Name ________________________ Class ____________ Date ____________

1-6 Reteaching

Absolute Value Equations and Inequalities

Solving absolute value equations require solving two equations separately. Recall that for a real number x, $|x|$ is the distance from zero to x on the number line. The equation $|x| = p$ means that either $x = p$ or $x = -p$ because both are p units from 0.

Problem

What is the solution set for the equation $|5x + 1| - 3 = 4$?

The first step in solving an absolute value equation is to isolate the absolute value on one side of the equal sign.

$|5x + 1| - 3 = 4$	
$|5x + 1| - 3 + 3 = 4 + 3$	Add 3 to each side.
$|5x + 1| = 7$	Simplify.

Next, rewrite the absolute value as two equations and solve each of them separately.

$5x + 1 = 7$	or	$5x + 1 = -7$	Definition of absolute value
$5x = 6$	or	$5x = -8$	Addition Property of Equality
$x = \frac{6}{5}$	or	$x = -\frac{8}{5}$	Division Property of Equality

Notice that the same operations are performed in the same order on each of the two equations. However, do not try to “simplify” the process by solving a single equation. This leads to errors.

The solutions are $x = \frac{6}{5}$ or $x = -\frac{8}{5}$. Check each solution in the original equation:

Check

$\left|5 \cdot \frac{6}{5} + 1\right| - 3 = 4$	$\left|5 \cdot \left(-\frac{8}{5}\right) + 1\right| - 3 = 4$
$|6 + 1| - 3 = 4$	$|-8 + 1| - 3 = 4$
$4 = 4$ ✓	$4 = 4$ ✓

Exercises

Solve each absolute value equation. Check your work.

1. $|2x - 3| - 4 = 3$ $x = -2$ or $x = 5$

2. $|3x - 6| + 1 = 13$ $x = 6$ or $x = -2$

1-6 Reteaching (continued)

Absolute Value Equations and Inequalities

To solve an absolute value inequality, keep in mind that $|x|$ is the distance from zero to x on the number line. So, if $|x| < p$, then x is less than p units from 0, so

$$|x| < p \quad \Rightarrow \quad -p < x < p.$$

And, if $|x| > p$, then x is greater than p units from 0, so

$$|x| > p \quad \Rightarrow \quad x < -p \text{ or } x > p.$$

In this case, we need to rewrite the absolute value inequality as two separate inequalities. Do not try to combine them into one inequality.

Problem

What is the solution set for the inequality $|2x + 3| > 11$?

Because the inequality is $>$, use $|x| > p \quad \Rightarrow \quad x < -p$ or $x > p$.

Begin by rewriting the absolute value as two equations and solve each of them separately.

$2x + 3 < -11$	or	$2x + 3 > 11$	Rewrite as a compound inequality.
$2x < -14$	or	$2x > 8$	Subtract 3 from each side.
$x > -7$	or	$x > 4$	Divide each side by 2.

The solution set is $x < -7$ or $x > 4$.

Exercises

Complete the steps to solve the inequality $\left|\frac{x}{2} - 4\right| \leq 3$.

3. $\boxed{-3} \leq \frac{x}{2} - 4 \leq \boxed{3}$ Rewrite as a compound inequality.

4. $\boxed{1} \leq \frac{x}{2} \leq \boxed{7}$ Add $\boxed{4}$ to each part.

5. $\boxed{2} \leq x \leq \boxed{14}$ Multiply each part by $\boxed{2}$.

6. What is the solution? $2 \leq x \leq 14$