3-4 Reteaching

Linear Programming

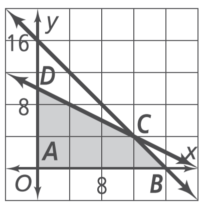
Problem

What point in the feasible region maximizes P for the objective function $P = 10x + 15y$? What point minimizes P?

Constraints $\begin{cases} x + y \leq 16 \\ 3x + 6y \leq 60 \\ x \geq 0 \\ y \geq 0 \end{cases}$

Step 1

Graph the constraints and shade the feasible region.



Step 2

Find the coordinates for each vertex of the region.

Step 3

Evaluate P at each vertex.

VERTEX	$P = 10x + 15y$
$A\,(0, 0)$	$P = 10(0) + 15(0) = 0$
$B\,(16, 0)$	$P = 10(16) + 15(0) = 160$
$C\,(12, 4)$	$P = 10(12) + 15(4) = 180$
$D\,(0, 10)$	$P = 10(0) + 15(10) = 150$

The maximum value of the objective function is 180. It occurs when $x = 12$ and $y = 4$.

The minimum value of the objective function is 0. It occurs when $x = 0$ and $y = 0$.

Exercises

Graph each system of constraints. Name all vertices. Then find the values of x and y that maximize or minimize the objective function.

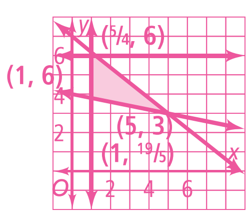
1. $\begin{cases} 5y + 4x \leq 35 \\ 5y + x \geq 20 \\ y \leq 6 \\ x \geq 1 \end{cases}$

$P = 8x + 2y$

vertices: $(1, 6)$, $\left(1, \frac{19}{5}\right)$, $\left(\frac{5}{4}, 6\right)$, $(5, 3)$

max: 46 at $(5, 3)$;

min: $\frac{78}{5}$ at $\left(1, \frac{19}{5}\right)$



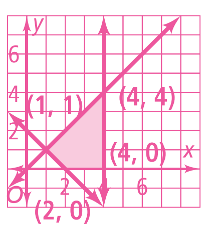
2. $\begin{cases} x + y \geq 2 \\ x \geq y \\ x \leq 4 \\ y \geq 0 \end{cases}$

$P = x + 3y$

vertices: $(4, 0)$, $(1, 1)$, $(2, 0)$, $(4, 4)$

max: 16 at $(4, 4)$;

min: 2 at $(2, 0)$



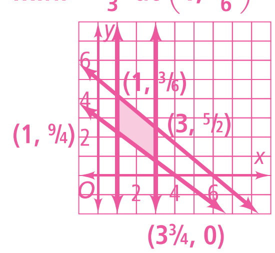
3. $\begin{cases} 3x + 4y \geq 12 \\ 5x + 6y \leq 30 \\ 1 \leq x \leq 3 \end{cases}$

$P = x - 2y$

vertices: $\left(1, \frac{9}{4}\right)$, $\left(1, \frac{25}{6}\right)$, $\left(3, \frac{3}{4}\right)$, $\left(3, \frac{5}{2}\right)$

max: $\frac{3}{2}$ at $\left(3, \frac{3}{4}\right)$;

min: $-\frac{22}{3}$ at $\left(1, \frac{25}{6}\right)$



3-4 Reteaching (continued)

Linear Programming

Problem

Your school band is selling calendars as a fundraiser. Wall calendars cost \$48 per case of 24. You sell them at \$7 per calendar. Pocket calendars cost \$30 per case of 40. You sell them at \$3 per calendar. You make a profit of \$120 per case of wall calendars and \$90 per case of pocket calendars. If the band can buy no more than 1000 total calendars and spend no more than \$1200, how can you maximize your profit if you sell every calendar? What is the maximum profit?

Relate Organize the information in a table.

	Wall Calendars	Pocket Calendars	Total
Number of Cases	x	y	
Number of Units	$24x$	$40y$	1000
Cost	$48x$	$30y$	1200
Profit	$120x$	$90y$	$120x + 90y$

Define Let x = number of cases of wall calendars

Let y = number of cases of pocket calendars

Write Use the information in the table and the definitions of x and y to write the constraints and the objective function. Simplify the inequalities if necessary.

$24x + 40y \leq 1000$ $\quad$ $48x + 30y \leq 1200$

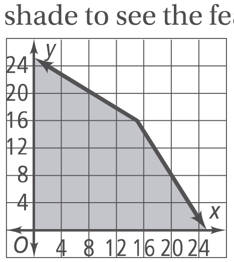
$3x + 5y \leq 125$ $\quad$ $8x + 5y \leq 200$

$$\begin{cases} 3x + 5y \leq 125 \\ 8x + 5y \leq 200 \\ x \geq 0,\ y \geq 0 \end{cases}$$

Objective function: $P = 120x + 90y$

Step 1

Graph the constraints and shade to see the feasible region.



Step 2

Find the coordinates for each vertex of the region.

$A(0, 0)$

$B(25, 0)$

$C(15, 16)$

$D(0, 25)$

Step 3

Evaluate the objective function using the vertex coordinates.

$P = 120(0) + 90(0) = 0$

$P = 120(25) + 90(0) = 3000$

$P = 120(15) + 90(16) = 3240$

$P = 120(0) + 90(25) = 2250$

You can maximize your profit by selling 15 cases of wall calendars and 16 cases of pocket calendars. The maximum profit is \$3240.

Exercises

4. Your band decides to sell the wall calendars for \$9 each.

a. How many of each type of calendar should you now buy to maximize your profit? **25 cases of wall calendars and no cases of pocket calendars**

b. What is the maximum profit? **\$4200**