

3-5 Reteaching

Systems With Three Variables

Problem

What is the solution of the system?

$$\begin{array}{l} \textcircled{1} \quad \left\{ \begin{array}{l} x + y + z = 6 \\ 2x - y + 3z = 9 \\ -x + 2y + 2z = 9 \end{array} \right. \end{array}$$

Use elimination. The equations are numbered to make the process easy to follow.

$$\begin{array}{l} \textcircled{1} \quad x + y + z = 6 \\ \textcircled{3} \quad -x + 2y + 2z = 9 \\ \hline \textcircled{4} \quad 3y + 3z = 15 \end{array} \quad \text{Pair the equations to eliminate } x.$$

$$\begin{array}{l} \textcircled{2} \quad 2x - y + 3z = 9 \\ \textcircled{3} \quad -x + 2y + 2z = 9 \end{array} \quad \text{Pair a different set of equations.}$$

$$\begin{array}{l} \textcircled{2} \quad 2x - y + 3z = 9 \\ \textcircled{3} \quad -2x + 4y + 4z = 18 \\ \hline \textcircled{5} \quad 3y + 7z = 27 \end{array} \quad \begin{array}{l} \text{Multiply equation 3 by 2 to eliminate } x. \\ \text{Then add the two equations.} \end{array}$$

$$\begin{array}{l} \textcircled{4} \quad 3y + 3z = 15 \\ \textcircled{5} \quad -3y + (-7z) = -27 \\ \hline \quad \quad -4z = -12 \\ \quad \quad z = 3 \end{array} \quad \begin{array}{l} \text{Equations 4 and 5 form a system.} \\ \text{Multiply equation 5 by } -1 \text{ and add to equation} \\ \text{4 to eliminate } y \text{ and solve for } z. \end{array}$$

$$\begin{array}{l} \textcircled{4} \quad 3y + 3(3) = 15 \\ \quad \quad 3y = 6 \\ \quad \quad y = 2 \end{array} \quad \text{Substitute } z = 3 \text{ into equation 4 and solve for } y.$$

$$\begin{array}{l} \textcircled{1} \quad x + 2 + 3 = 6 \\ \quad \quad x = 1 \end{array} \quad \text{Substitute the values of } y \text{ and } z \text{ into one of the} \\ \text{original equations. Solve for } x.$$

The solution is $(1, 2, 3)$.

Exercises

Solve each system by elimination. Check your answers.

$$\begin{array}{lll} 1. \left\{ \begin{array}{l} 2x - y + 2z = 10 \\ 4x + 2y - 5z = 10 \\ x - 3y + 5z = 8 \end{array} \right. & 2. \left\{ \begin{array}{l} x - y + z = 6 \\ 2x + 3y + 2z = 2 \\ 3x + 5y + 4z = 4 \end{array} \right. & 3. \left\{ \begin{array}{l} 6x - 4y + 5z = 31 \\ 5x + 2y + 2z = 13 \\ x + y + z = 2 \end{array} \right. \\ \text{(4, 2, 2)} & \text{(2, -2, 2)} & \text{(3, -2, 1)} \end{array}$$

$$\begin{array}{lll} 4. \left\{ \begin{array}{l} 3x + y + z = 2 \\ 4x - 2y + 3z = -4 \\ 2x + 2y + 2z = 8 \end{array} \right. & 5. \left\{ \begin{array}{l} 5x + 2y + z = 5 \\ 3x - 3y - 3z = 9 \\ x + 2y + 4z = 6 \end{array} \right. & 6. \left\{ \begin{array}{l} x + y + z = -1 \\ 4x + 3y + 2z = -10 \\ 2x - 4y - 2z = -6 \end{array} \right. \\ \text{(-1, 3, 2)} & \text{(2, -4, 3)} & \text{(-3, -2, 4)} \end{array}$$

3-5 Reteaching (continued)

Systems With Three Variables

Problem

What is the solution of the system? Use substitution.

$$\begin{cases} \textcircled{1} & x + 3y - 2z = 19 \\ \textcircled{2} & 4x - 2y + 3z = 8 \\ \textcircled{3} & -3x + 2y + 2z = 15 \end{cases}$$

Step 1 Choose an equation that can be solved easily for one variable. Choose equation 1 and solve for x .

$$x = -3y + 2z + 19$$

Step 2 Substitute the expression for x into equations 2 and 3 and simplify.

$$\begin{array}{rcl} 4(-3y + 2z + 19) - 2y + 3z = 8 & & -3(-3y + 2z + 19) + 2y + 2z = 15 \\ -12y + 8z + 76 - 2y + 3z = 8 & & 9y - 6z - 57 + 2y + 2z = 15 \\ \textcircled{4} \quad -14y + 11z = -68 & & \textcircled{5} \quad 11y - 4z = 72 \end{array}$$

Step 3 Write the two new equations as a system. Solve for y and z .

$$\begin{cases} \textcircled{4} & -14y + 11z = -68 \\ \textcircled{5} & 11y - 4z = 72 \end{cases}$$

$$\begin{array}{rcl} -56y + 44z = -272 & \text{Multiply } \textcircled{4} \text{ by 4.} & -14y + 11z = -68 \\ 121y - 44z = 792 & \text{Multiply } \textcircled{5} \text{ by 11.} & -14(8) + 11z = -68 \\ \hline 65y = 520 & & 11z = 44 \\ y = 8 & & z = 4 \end{array}$$

Substitute $y = 8$ into $\textcircled{4}$.

Step 4 Use one of the original equations to solve for x .

$$x + 3y - 2z = 19 \quad \text{Substitute } y = 8 \text{ and } z = 4 \text{ into } \textcircled{1}.$$

$$x + 3(8) - 2(4) = 19$$

$$x = 3$$

The solution of the system is (3, 8, 4).

Exercises

Solve each system by substitution. Check your answers.

<p>7. $\begin{cases} -4x + 3y - 2z = 7 \\ 2x - 2y + 3z = 15 \\ -x + 2y - 2z = -6 \end{cases}$</p> <p>(-2, 7, 11)</p>	<p>8. $\begin{cases} -2x - y + 5z = -5 \\ 3x + 2y - 7z = 10 \\ -2x - 3y + 6z = -12 \end{cases}$</p> <p>(3, 4, 1)</p>	<p>9. $\begin{cases} 2x - y + 3z = 15 \\ 3x - 4y - 2z = 7 \\ 2x + 2y + 5z = 22 \end{cases}$</p> <p>(5, 1, 2)</p>
<p>10. $\begin{cases} -2x + 2y - z = -1 \\ x + 2y + 2z = 11 \\ 4x - 3y + 2z = 4 \end{cases}$</p> <p>(1, 2, 3)</p>	<p>11. $\begin{cases} x - 3y + 2z = 7 \\ 2x - 5y - 4z = -1 \\ x + 4y - 6z = -2 \end{cases}$</p> <p>(6, 1, 2)</p>	<p>12. $\begin{cases} 3x - y - 3z = -13 \\ 2x + 4y - 5z = -5 \\ -5x + 2y + z = 3 \end{cases}$</p> <p>(2, 4, 5)</p>