

3-6

Reteaching

Solving Systems Using Matrices

Problem

How can you represent the system of equations with a matrix?
$$\begin{cases} 4x - 3y + 5z = -13 \\ x + 3y = 3 \\ -2x + 4y + 3z = 17 \end{cases}$$

Step 1 Write each equation in the same variable order. Line up the like variables. Write in variables that have a coefficient of 0.

$$\begin{cases} 4x - 3y + 5z = -13 \\ x + 3y + 0z = 3 \\ -2x + 4y + 3z = 17 \end{cases}$$

Step 2 Write the matrix using the coefficients and constants. Remember to enter a 1 for variables with no numeric coefficient.

$$\left[\begin{array}{ccc|c} 4 & -3 & 5 & -13 \\ 1 & 3 & 0 & 3 \\ -2 & 4 & 3 & 17 \end{array} \right]$$

Exercises

Write a matrix to represent each system.

1.
$$\begin{cases} 2x + y = -3 \\ 3y = 5 \end{cases}$$

$$\left[\begin{array}{cc|c} 2 & 1 & -3 \\ 0 & 3 & 5 \end{array} \right]$$

2.
$$\begin{cases} 3x - 5y + 2z = 9 \\ 4x + 7y + z = 3 \\ 2x - z = 12 \end{cases}$$

$$\left[\begin{array}{ccc|c} 3 & -5 & 2 & 9 \\ 4 & 7 & 1 & 3 \\ 2 & 0 & -1 & 12 \end{array} \right]$$

3.
$$\begin{cases} 5x - y + 3z = 2 \\ 3y + 2z = 6 \\ 4x + 3y + z = 1 \end{cases}$$

$$\left[\begin{array}{ccc|c} 5 & -1 & 3 & 2 \\ 0 & 3 & 2 & 6 \\ 4 & 3 & 1 & 1 \end{array} \right]$$

4.
$$\begin{cases} 2x - z = 3 \\ 5y + 4z = -5 \\ -x + 2y = 1 \end{cases}$$

$$\left[\begin{array}{ccc|c} 2 & 0 & -1 & 3 \\ 0 & 5 & 4 & -5 \\ -1 & 2 & 0 & 1 \end{array} \right]$$

5.
$$\begin{cases} z = 6 \\ x + y = 2 \\ 3x - 2y - 5z = 10 \end{cases}$$

$$\left[\begin{array}{ccc|c} 0 & 0 & 1 & 6 \\ 1 & 1 & 0 & 2 \\ 3 & -2 & -5 & 10 \end{array} \right]$$

6.
$$\begin{cases} 2z - 5x + 3y = 4 \\ -y + 2x + 4z = -2 \\ 3x + z - 2y = -5 \end{cases}$$

$$\left[\begin{array}{ccc|c} -5 & 3 & 2 & 4 \\ 2 & -1 & 4 & -2 \\ 3 & -2 & 1 & -5 \end{array} \right]$$

3-6 **Reteaching** (continued)

Solving Systems Using Matrices

Problem

What is the solution of the system? $\begin{cases} 2x + 5y = 5 \\ -x + 2y = -7 \end{cases}$

Step 1 Write the matrix for the system.

$$\left[\begin{array}{cc|c} 2 & 5 & 5 \\ -1 & 2 & -7 \end{array} \right]$$

Step 2 Multiply Row 2 by 2. Add to Row 1. Replace Row 1 with the sum. Write the new matrix.

$$\begin{array}{ccc} 2 & 5 & 5 \\ +2(-1 & 2 & -7) \\ \hline 0 & 9 & -9 \end{array} \quad \left[\begin{array}{cc|c} 0 & 9 & -9 \\ -1 & 2 & -7 \end{array} \right]$$

Step 3 Divide Row 1 by 9. Write the new matrix.

$$\frac{1}{9}(0 \quad 9 \quad -9) \quad \left[\begin{array}{cc|c} 0 & 1 & -1 \\ -1 & 2 & -7 \end{array} \right]$$

Step 4 Multiply Row 1 by -2 . Add to Row 2. Replace Row 2 with the sum. Write the new matrix.

$$\begin{array}{ccc} -2(0 & 1 & -1) \\ + -1 & 2 & -7 \\ \hline -1 & 0 & -5 \end{array} \quad \left[\begin{array}{cc|c} 0 & 1 & -1 \\ -1 & 0 & -5 \end{array} \right]$$

Step 5 Multiply Row 2 by -1 . Write the new matrix.

$$-1(-1 \quad 0 \quad -5) \quad \left[\begin{array}{cc|c} 0 & 1 & -1 \\ 1 & 0 & 5 \end{array} \right]$$

This matrix is equivalent to the system $\begin{cases} y = -1 \\ x = 5 \end{cases}$. The solution is $(5, -1)$.

Exercises

Solve each system of equations using a matrix.

7. $\begin{cases} 4x + 3y = 6 \\ -x - y = -1 \end{cases}$
 $(3, -2)$

8. $\begin{cases} 6x + y = -2 \\ -x + 3y = 13 \end{cases}$
 $(-1, 4)$

9. $\begin{cases} 3x + 2y = -4 \\ -4x - 3y = 7 \end{cases}$
 $(2, -5)$