Name _____ Class _____ Date _____

4-3 Reteaching

Modeling With Quadratic Functions

Three non-collinear points, no two of which are in line vertically, are on the graph of exactly one quadratic function.

Problem

A parabola contains the points $(0, -2)$, $(-1, 5)$, and $(2, 2)$. What is the equation of this parabola in standard form?

If the parabola $y = ax^2 + bx + c$ passes through the point (x, y), the coordinates of the point must satisfy the equation of the parabola. Substitute the (x, y) values into $y = ax^2 + bx + c$ to write a system of equations.

First, use the point $(0, -2)$.	$y = ax^2 + bx + c$	Write the standard form.
	$-2 = a(0)^2 + b(0) + c$	Substitute.
	$-2 = c$	Simplify.
Use the point $(-1, 5)$ next.	$5 = a(-1)^2 + b(-1) + c$	Substitute.
	$5 = a - b + c$	Simplify.
Finally, use the point $(2, 2)$.	$2 = a(2)^2 + b(2) + c$	Substitute.
	$2 = 4a + 2b + c$	Simplify.

Because $c = -2$, the resulting system has two variables. Simplify the equations above.

$$a - b = 7$$
$$4a + 2b = 4$$

Use elimination to solve the system and obtain $a = 3$, $b = -4$, and $c = -2$. Substitute these values into the standard form $y = ax^2 + bx + c$.

The equation of the parabola that contains the given points is $y = 3x^2 - 4x - 2$.

Exercises

Find an equation in standard form of the parabola passing through the given points.

1. $(0, -1), (1, 5), (-1, -5)$ $y = x^2 + 5x - 1$
2. $(0, 4), (-1, 9), (2, 0)$ $y = x^2 - 4x + 4$
3. $(0, 1), (1, 4), (3, 22)$ $y = 2x^2 + x + 1$
4. $(1, -1), (-2, 20), (2, 0)$ $y = 2x^2 - 5x + 2$
5. $(-1, -5), (0, -1), (2, 1)$ $y = -x^2 + 3x - 1$
6. $(1, 3), (-2, -3), (-1, 3)$ $y = -2x^2 + 5$

4-3 Reteaching (continued)

Modeling With Quadratic Functions

Problem

A soccer player kicks a ball off the top of a building. His friend records the height of the ball at each second. Some of her data appears in the table.

a. What is a quadratic model for these data?

b. Use the model to complete the table.

Time (s)	Height (ft)
0	112
1	192
2	
3	
4	
5	192
6	
7	

Use the points (0, 112), (1, 192), and (5, 192) to find the quadratic model. Substitute the (t, h) values into $h = at^2 + bt + c$ to write a system of equations.

$(0, 112)$: $112 = a(0)^2 + b(0) + c$ $\quad c = 112$

$(1, 192)$: $192 = a(1)^2 + b(1) + c$ $\quad a + b + c = 192$

$(5, 192)$: $192 = a(5)^2 + b(5) + c$ $\quad 25a + 5b + c = 192$

Use $c = 112$ and simplify the equations to obtain a system with just two variables.

$$a + b = 80$$

$$25a + 5b = 80$$

Use elimination to solve the system. The quadratic model for the data is

$$h = -16t^2 + 96t + 112$$

Now use this equation to complete the table for the t-values 2, 3, 4, 6, and 7.

Time (s)	Height (ft)
0	112
1	192
2	240
3	256
4	240
5	192
6	112
7	0

$t = 2$: $h = -16(2)^2 + 96(2) + 112 = -64 + 192 + 112 = 240$

$t = 3$: $h = -16(3)^2 + 96(3) + 112 = -144 + 288 + 112 = 256$

$t = 4$: $h = -16(4)^2 + 96(4) + 112 = -256 + 384 + 112 = 240$

$t = 6$: $h = -16(6)^2 + 96(6) + 112 = -576 + 576 + 112 = 112$

$t = 7$: $h = -16(7)^2 + 96(7) + 112 = -784 + 672 + 112 = 0$

Exercise

7. The number n of Brand X shoes in stock at the beginning of month t in a store follows a quadratic model. In January ($t = 1$), there are 36 pairs of shoes; in March ($t = 3$), there are 52 pairs; and in September, there are also 52 pairs.

a. What is the quadratic model for the number n of pairs of shoes at the beginning of month t? $n = -t^2 + 12t + 25$

b. How many pairs are in stock in June? 61