

4-7 Reteaching

The Quadratic Formula

You can solve some quadratic equations by factoring or completing the square. You can solve any quadratic equation $ax^2 + bx + c = 0$ by using the Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Notice the \pm symbol in the formula. Whenever $b^2 - 4ac$ is not zero, the Quadratic Formula will result in two solutions.

Problem

What are the solutions for $2x^2 + 3x = 4$? Use the Quadratic Formula.

$$2x^2 + 3x - 4 = 0$$

Write the equation in standard form: $ax^2 + bx + c = 0$

$$a = 2; b = 3; c = -4$$

a is the coefficient of x^2 , b is the coefficient of x , c is the constant term.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Write the Quadratic Formula.

$$= \frac{-3 \pm \sqrt{(3)^2 - 4(2)(-4)}}{2(2)}$$

Substitute 2 for a , 3 for b , and -4 for c .

$$= \frac{-3 \pm \sqrt{41}}{4}$$

Simplify.

$$= \frac{-3 + \sqrt{41}}{4} \text{ or } \frac{-3 - \sqrt{41}}{4}$$

Write the solutions separately.

Check your results on your calculator. Replace x in the original equation with $\frac{-3 + \sqrt{41}}{4}$ and $\frac{-3 - \sqrt{41}}{4}$. Both values for x give a result of 4. The solutions check.

$(-3 + \sqrt{(41)})/4 \rightarrow X$
 $.8507810594$
 $2X^2 + 3X = 4$

$(-3 - \sqrt{(41)})/4 \rightarrow X$
 -2.350781059
 $2X^2 + 3X = 4$

Exercises

What are the solutions for each equation? Use the Quadratic Formula.

1. $-x^2 + 7x - 3 = 0$ $\frac{7 + \sqrt{37}}{2}$ or $\frac{7 - \sqrt{37}}{2}$

2. $x^2 + 6x = 10$ $-3 + \sqrt{19}$ or $-3 - \sqrt{19}$

3. $2x^2 = 4x + 3$ $\frac{2 + \sqrt{10}}{2}$ or $\frac{2 - \sqrt{10}}{2}$

4. $4x^2 + 81 = 36x$ $\frac{9}{2}$

5. $2x^2 + 1 = 5 - 7x$ -4 or $\frac{1}{2}$

6. $6x^2 - 10x + 3 = 0$ $\frac{5 + \sqrt{7}}{6}$ or $\frac{5 - \sqrt{7}}{6}$

4-7 Reteaching (continued)

The Quadratic Formula

There are three possible outcomes when you take the square root of a real number n :

$$n \begin{cases} > 0 & \rightarrow & \text{two real values (one positive and one negative)} \\ = 0 & \rightarrow & \text{one real value (0)} \\ < 0 & \rightarrow & \text{no real values} \end{cases}$$

Now consider the quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. The value under the radical symbol determines the number of real solutions that exist for the equation $ax^2 + bx + c = 0$:

$$b^2 - 4ac \begin{cases} > 0 & \rightarrow & \text{two real solutions} \\ = 0 & \rightarrow & \text{one real solution} \\ < 0 & \rightarrow & \text{no real solutions} \end{cases}$$

The value under the radical, $b^2 - 4ac$, is called the **discriminant**.

Problem

What is the number of real solutions of $-3x^2 + 7x = 2$?

$$\begin{aligned} -3x^2 + 7x &= 2 \\ -3x^2 + 7x - 2 &= 0 && \text{Write in standard form.} \\ a = -3, b = 7, c = -2 &&& \text{Find the values of } a, b, \text{ and } c. \\ b^2 - 4ac &&& \text{Write the discriminant.} \\ (7)^2 - 4(-3)(-2) &&& \text{Substitute for } a, b, \text{ and } c. \\ 49 - 24 &&& \text{Simplify.} \\ 25 &&& \end{aligned}$$

The discriminant, 25, is positive. The equation has two real roots.

Exercises

What is the value of the discriminant and what is the number of real solutions for each equation?

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| 7. $x^2 + x - 42 = 0$
169; two | 8. $-x^2 + 13x - 40 = 0$
9; two | 9. $x^2 + 2x + 5 = 0$
-16; none |
| 10. $x^2 = 18x - 81$
0; one | 11. $-x^2 + 7x + 44 = 0$
225; two | 12. $\frac{1}{4}x^2 - 5x + 25 = 0$
0; one |
| 13. $2x^2 + 7 = 5x$
-31; none | 14. $4x^2 + 25x = 21$
961; two | 15. $x^2 + 5 = 3x$
-11; none |
| 16. $\frac{1}{9}x^2 = 4x - 36$
0; one | 17. $\frac{1}{2}x^2 + 2x + 3 = 0$
-2; none | 18. $\frac{1}{6}x^2 = 2x + 18$
16; two |