Name _____ Class _____ Date _____

4-8 Reteaching

Complex Numbers

- A *complex number* consists of a real part and an imaginary part. It is written in the form $a + bi$, where a and b are real numbers.
- $i = \sqrt{-1}$ and $i^2 = (\sqrt{-1})(\sqrt{-1}) = -1$
- When adding or subtracting complex numbers, combine the real parts and then combine the imaginary parts.
- When multiplying complex numbers, use the Distributive Property or FOIL.

Problem

What is $(3 - i) + (2 + 3i)$?

$(3 - i) + (2 + 3i)$

$= \textcircled{3} - \boxed{i} + \textcircled{2} + \boxed{3i}$ Circle real parts. Put a square around imaginary parts.

$= (3 + 2) + (-1 + 3)i$ Combine.

$= 5 + 2i$ Simplify.

Problem

What is the product $(7 - 3i)(-4 + 9i)$?

Use FOIL to multiply:

$(7 - 3i)(-4 + 9i) = 7(-4) + 7(9i) + (-3i)(-4) + (-3i)(9i)$

$(7 - 3i)(-4 + 9i)$

$= -28 + 63i + 12i - 27i^2$

First $= 7(-4)$

$= -28 + 75i - 27i^2$

Outer $= 7(9i)$

You can simplify the expression by substituting -1 for i^2.

Inner $= (-3i)(-4)$

$(7 - 3i)(-4 + 9i) = -28 + 75i - 27(-1)$

Last $= (-3i)(9i)$

$= -1 + 75i$

Exercises

Simplify each expression.

1. $2i + (-4 - 2i)$
   -4
2. $(3 + i)(2 + i)$
   $5 + 5i$
3. $(4 + 3i)(1 + 2i)$
   $-2 + 11i$
4. $3i(1 - 2i)$
   $6 + 3i$
5. $3i(4 - i)$
   $3 + 12i$
6. $3 - (-2 + 3i) + (-5 + i)$
   $-2i$
7. $4i(6 - 2i)$
   $8 + 24i$
8. $(5 + 6i) + (-2 + 4i)$
   $3 + 10i$
9. $9(11 + 5i)$
   $99 + 45i$

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4-8 Reteaching (continued)

Complex Numbers

- The *complex conjugate* of a complex number $a + bi$ is the complex number $a - bi$.
- $(a + bi)(a - bi) = a^2 + b^2$
- To divide complex numbers, use complex conjugates to simplify the denominator.

Problem

What is the quotient $\frac{4 + 5i}{2 - i}$?

$\frac{4 + 5i}{2 - i}$	The complex conjugate of $2 - i$ is $2 + i$.
$= \frac{4 + 5i}{2 - i} \cdot \frac{2 + i}{2 + i}$	Multiply both numerator and denominator by $2 + i$.
$= \frac{8 + 4i + 10i + 5i^2}{(2 - i)(2 + i)}$	Use FOIL to multiply the numerators.
$= \frac{8 + 4i + 10i + 5i^2}{2^2 + 1^2}$	Simplify the denominator. $(a + bi)(a - bi) = a^2 + b^2$
$= \frac{8 + 14i + 5(-1)}{4 + 1}$	Substitute -1 for i^2.
$= \frac{3 + 14i}{5}$	Simplify.
$= \frac{3}{5} + \frac{14}{5}i$	Write as a complex number $a + bi$.

Exercises

Find the complex conjugate of each complex number.

10. $1 - 2i$ $1 + 2i$

11. $3 + 5i$ $3 - 5i$

12. i $-i$

13. $3 - i$ $3 + i$

14. $2 + 3i$ $2 - 3i$

15. $-5 - 2i$ $-5 + 2i$

Write each quotient as a complex number.

16. $\frac{3i}{1 - 2i}$ $-\frac{6}{5} + \frac{3}{5}i$

17. $\frac{6}{3 + 5i}$ $\frac{9}{17} - \frac{15}{17}i$

18. $\frac{2 + 2i}{i}$ $2 - 2i$

19. $\frac{2 + 5i}{3 - i}$ $\frac{1}{10} + \frac{17}{10}i$

20. $\frac{-4 - i}{2 + 3i}$ $-\frac{11}{13} + \frac{10}{13}i$

21. $\frac{6 + i}{-5 - 2i}$ $-\frac{32}{29} + \frac{7}{29}i$