Name _____ Class _____ Date _____

5-2 Reteaching

Polynomials, Linear Factors, and Zeros

The Factor Theorem tells you that if you know the zeros of a polynomial function, you can write the polynomial.

Factor Theorem

The expression $x - a$ is a factor of a polynomial if and only if the value a is a zero of the related polynomial function.

Problem

What is a cubic polynomial function in standard form with zeros 0, 4, and -2?

Each zero (a) is part of a linear factor of the polynomial, so you can write each factor as $(x - a)$.

$(x - a_1)(x - a_2)(x - a_3)$	Set up the cubic polynomial factors.
$a_1 = 0, a_2 = 4, a_3 = -2$	Assign the zeros.
$(x - 0)(x - 4)[x - (-2)]$	Substitute the zeros into the factors.
$f(x) = x(x - 4)(x + 2)$	Write the polynomial function in factored form.
$f(x) = x(x^2 - 2x - 8)$	Multiply $(x - 4)(x + 2)$.
$f(x) = x^3 - 2x^2 - 8x$	Multiply by x using the Distributive Property.

The polynomial function written in standard form is $f(x) = x^3 - 2x^2 - 8x$.

Exercises

Write a polynomial function in standard form with the given zeros.

1. $5, -1, 3$ $f(x) = x^3 - 7x^2 + 7x + 15$
2. $1, 7, -5$ $f(x) = x^3 - 3x^2 - 33x + 35$
3. $-1, 1, -6$ $f(x) = x^3 + 6x^2 - x - 6$
4. $2, -2, -3$ $f(x) = x^3 + 3x^2 - 4x - 12$
5. $2, 1, 3$ $f(x) = x^3 - 6x^2 + 11x - 6$
6. $2, 3, -3, -1$ $f(x) = x^4 - x^3 - 11x^2 + 9x + 18$
7. $0, -8, 2$ $f(x) = x^3 + 6x^2 - 16x$
8. $-10, 0, 2$ $f(x) = x^3 + 8x^2 - 20x$
9. $-2, 2, -\frac{3}{2}$ $f(x) = x^3 + \frac{3}{2}x^2 - 4x - 6$
10. $-1, \frac{2}{3}$ $f(x) = x^2 + \frac{1}{3}x - \frac{2}{3}$

Name _____ Class _____ Date _____

5-2 Reteaching (continued)

Polynomials, Linear Factors, and Zeros

You can use a polynomial function to find the minimum or maximum value of a function that satisfies a given set of conditions.

Problem

Your school wants to put in a swimming pool. The school wants to maximize the volume while keeping the sum of the dimensions at 40 ft. If the length must be 2 times the width, what should each dimension be?

Step 1 First, define a variable x. Let $x =$ the width of the pool.

Step 2 Determine the length and depth of the pool using the information in the problem.

The length must be 2 times the width, so length $= 2x$.

The length plus width plus depth must equal 40 ft, so depth $= 40 - x - 2x = 40 - 3x$.

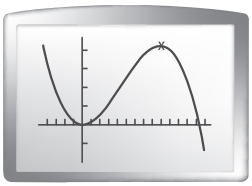
Step 3 Create a polynomial in standard form using the volume formula

$V =$ length $\cdot$ width $\cdot$ depth

$= 2x(x)(40 - 3x)$

$= -6x^3 + 80x^2$

Step 4 Graph the polynomial function. Use the MAXIMUM feature. The maximum volume is 2107 ft^3 at a width of 8.9 ft.



Maximum
X = 8.8888882 Y = 2106.9959

Step 5 Evaluate the remaining dimensions: width $= x \approx 8.9$ ft

length $= 2x \approx 17.8$ ft

depth $= 40 - 3x \approx 13.3$ ft

Exercises

11. Find the dimensions of the swimming pool if the sum must be 50 ft and the length must be 3 times the depth. **depth = 8.3 ft, length = 24.9 ft, width = 16.8 ft**

12. Find the dimensions of the swimming pool if the sum must be 40 ft and the depth must be one tenth of the length. **length = 24.2 ft, depth = 2.42 ft, width = 13.4 ft**

13. Find the dimensions of the swimming pool if the sum must be 60 ft and the length and width are equal. **length = 20 ft, width = 20 ft, depth = 20 ft**