

5-3 Reteaching

Solving Polynomial Equations

Problem

What are the real or imaginary solutions of the polynomial equation

$$2x^3 + 16 = 0?$$

$$2x^3 + 16 = 0$$

$$2(x^3 + 8) = 0$$

$$2(x + 2)(x^2 - 2x + 4) = 0$$

$$x + 2 = 0 \text{ or } x^2 - 2x + 4 = 0$$

$$x = -2 \text{ or } x = \frac{2 \pm \sqrt{4 - 4(1)(4)}}{2(1)}$$

$$x = -2 \text{ or } x = \frac{2 \pm 2i\sqrt{3}}{2}$$

$$x = -2 \text{ or } x = 1 \pm i\sqrt{3}$$

Factor out the GCF. In this case, it is 2.

Factor the remaining cubic expression.

Use the Zero-Product Property.

Solve each equation for x . Use the Quadratic Formula when necessary.

Simplify.

The solutions are -2 and $1 \pm i\sqrt{3}$.

Exercises

Find the real or imaginary solutions of each polynomial equation.

1. $x^3 - 8 = 0$

$$2, -1 \pm i\sqrt{3}$$

3. $x^4 - x^2 - 72 = 0$

$$-3, 3, -2i\sqrt{2}, 2i\sqrt{2}$$

5. $x^4 - 27x = 0$

$$0, 3, \frac{-3 \pm 3i\sqrt{3}}{2}$$

7. $7x^4 = -28x^2 - 21$

$$-i, i, -i\sqrt{3}, i\sqrt{3}$$

9. $8x^3 + 27 = 0$

$$-\frac{3}{2}, \frac{3 \pm 3i\sqrt{3}}{4}$$

11. $2x^4 + 16x^2 = 40$

$$-\sqrt{2}, \sqrt{2}, -i\sqrt{10}, i\sqrt{10}$$

13. $9x^4 - 25 = 0$

$$-\frac{\sqrt{15}}{3}, \frac{\sqrt{15}}{3}, -i\frac{\sqrt{15}}{3}, i\frac{\sqrt{15}}{3}$$

15. $x^4 + 5x^2 = -4$

$$-i, i, -2i, 2i$$

17. $2x^3 + 16 = 0$

$$-2, 1 \pm i\sqrt{3}$$

2. $4x^3 + 4 = 0$

$$-1, \frac{1 \pm i\sqrt{3}}{2}$$

4. $x^4 + 9x^2 = -20$

$$-2i, 2i, -i\sqrt{5}, i\sqrt{5}$$

6. $8x^3 = -1$

$$-\frac{1}{2}, \frac{1 \pm i\sqrt{3}}{4}$$

8. $x^3 = 64$

$$4, -2 \pm 2i\sqrt{3}$$

10. $x^4 - 7x^2 = -12$

$$-2, 2, -\sqrt{3}, \sqrt{3}$$

12. $2x^4 - 16x = 0$

$$0, 2, -1 \pm i\sqrt{3}$$

14. $2x^4 - x^2 = 3$

$$-\frac{\sqrt{6}}{2}, \frac{\sqrt{6}}{2}, -i, i$$

16. $x^4 - 7x^2 - 8 = 0$

$$-2\sqrt{2}, 2\sqrt{2}, -i, i$$

18. $x^4 - 5x^2 - 24 = 0$

$$-2\sqrt{2}, 2\sqrt{2}, -i\sqrt{3}, i\sqrt{3}$$

5-3 **Reteaching** (continued)

Solving Polynomial Equations

Problem

You have a brother and a sister. Your brother is 3 years older than you. Your sister is 2 years older than your brother. The product of all three ages is 3744. How old are you and your siblings?

Step 1 Define the variables.

Let your age = x .

Your brother's age = $x + 3$.

Your sister's age = $x + 3 + 2 = x + 5$.

Step 2 Write an equation.

Because the product of all three ages equals 3744, $x(x + 3)(x + 5) = 3744$.

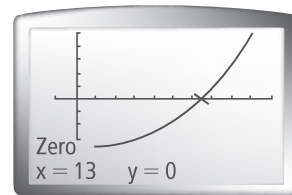
Step 3 Rewrite the equation in $P(x) = 0$ form.

$$(x^2 + 3x)(x + 5) = 3744$$

$$x^3 + 3x^2 + 5x^2 + 15x = 3744$$

$$x^3 + 8x^2 + 15x - 3744 = 0$$

Step 4 Using a graphing calculator, graph the equation. Use the Zero feature to solve for x .



Step 5 Once you have the value of x (your age), you can solve for the other ages. Since $x = 13$, your brother's age is $x + 3 = 16$. Your sister's age is $x + 5 = 18$.

Exercises

- 19.** A slice of wood 3 in. thick is cut off a cube of wood. The remaining solid has a volume of 320 in.^2 . What are the dimensions of the original block of wood?
8 in. \times 8 in. \times 8 in.
- 20.** The water level in a rectangular fish tank is 4 in. from the top. The depth of the water is the same as the width of the tank, which is half of its length. The volume of the water in the tank is 4394 in.^3 . What is the volume of the fish tank?
5746 in.³