5-5 Reteaching

Theorems About Roots of Polynomials Equations

Problem

What are the rational roots of $6x^4 + 29x^3 + 40x^2 + 7x - 12 = 0$?

Step 1 Determine the factors of the constant term and the factors of the leading coefficient.

constant term: 12 factors: $\pm1, \pm2, \pm3, \pm4, \pm6, \pm12$

leading coefficient: 6 factors: $\pm1, \pm2, \pm3, \pm6$

Step 2 Find all the possible roots by dividing the factors of the constant term by the factors of the leading coefficient. $\pm1, \pm2, \pm3, \pm4, \pm6, \pm12, \pm\frac{1}{6}, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}$

Step 3 Substitute each possible root into the polynomial until you find one that causes the polynomial to equal zero. This is one rational root.

Test $-\frac{3}{2}$: $6\left(-\frac{3}{2}\right)^4 + 29\left(-\frac{3}{2}\right)^3 + 40\left(-\frac{3}{2}\right)^2 + 7\left(-\frac{3}{2}\right) - 12 = 0$ $-\frac{3}{2}$ is a rational root.

Step 4 Factor the polynomial by synthetic division using the first rational root as the divisor.

$-\frac{3}{2}$	6	29	40	7	−12
		−9	−30	−15	12
	6	20	10	−8	0

Step 5 If the dividend is a second-degree polynomial, factor to find any additional rational roots. If the dividend does not factor, there are no additional rational roots. If the dividend is greater than a second-degree polynomial, repeat Steps 1–4 until the dividend is a second-degree polynomial.

$-\frac{4}{3}$	6	20	10	−8
		−8	−16	8
	6	12	−6	0

$6x^2 + 12x - 6 = 0$ does not factor. The rational roots of $6x^4 + 29x^3 + 40x^2 + 7x - 12$ are $-\frac{3}{2}$ and $-\frac{4}{3}$.

Exercises

Find all rational roots for $P(x) = 0$.

1. $P(x) = x^3 - x^2 - 8x + 12$ **−3, 2**

2. $P(x) = x^4 - 49x^2$ **0, 7, −7**

3. $P(x) = 2x^3 - 7x^2 - 21x + 54$ **−3, 2, $\frac{9}{2}$**

4. $P(x) = x^4 - 2x^3 - 3$ **−1**

5-5 Reteaching (continued)

Theorems About Roots of Polynomials Equations

Problem

What is a third-degree polynomial function $y = P(x)$ with rational coefficients so that $P(x) = 0$ has roots -4 and $2 \pm 3i$?

Roots: $-4, 2 - 3i, 2 + 3i$	Because $2 - 3i$ is a root, its complex conjugate $2 + 3i$ is also a root.
$(x + 4)[x - (2 - 3i)][x - (2 + 3i)]$	Write the factored form of the polynomial.
$(x + 4)[x^2 - x(2 + 3i) - x(2 - 3i) + (2 - 3i)(2 + 3i)]$	Multiply the factors.
$(x + 4)[x^2 - 2x - 3ix - 2x + 3ix + 4 + 6i - 6i - 9i^2]$	Multiply.
$(x + 4)[x^2 - 4x + 4 - 9i^2]$	Simplify.
$(x + 4)(x^2 - 4x + 13)$	Combine like terms.
$x^3 + 4x^2 - 4x^2 - 16x + 13x + 52$	Multiply.
$x^3 - 3x + 52$	Combine like terms.

A third-degree polynomial function with rational coefficients so that $P(x) = 0$ has roots -4 and $2 \pm 3i$ is $P(x) = x^3 - 3x + 52$.

Exercises

Write a third-degree polynomial function $y = P(x)$ with rational coefficients so that $P(x) = 0$ has the given roots.

5. $1, 2 - i$ **$P(x) = x^3 - 5x^2 + 9x - 5$**

6. $5 + 2i, -2$ **$P(x) = x^3 - 8x^2 + 9x + 58$**

7. $3, 6 + i$ **$P(x) = x^3 - 15x^2 + 73x - 111$**

8. $-4, \sqrt{2}$ **$P(x) = x^3 + 4x^2 - 2x - 8$**

9. $2 - \sqrt{3}, -1$ **$P(x) = x^3 - 3x^2 - 3x + 1$**

10. $0, 3 - \sqrt{3}$ **$P(x) = x^3 - 6x^2 + 6x$**

11. $3i, 7$ **$P(x) = x^3 - 7x^2 + 9x - 63$**

12. $2 + \sqrt{5}, 3$ **$P(x) = x^3 - 7x^2 + 11x + 3$**

13. $-3, i$ **$P(x) = x^3 + 3x^2 + x + 3$**

14. $1 - i, 8$ **$P(x) = x^3 - 10x^2 + 18x - 16$**

15. $1, 5i$ **$P(x) = x^3 - x^2 + 25x - 25$**

16. $2, 4 + i$ **$P(x) = x^3 - 10x^2 + 33x - 34$**

17. $3, -4i$ **$P(x) = x^3 - 3x^2 + 16x - 48$**

18. $0, 2 - i$ **$P(x) = x^3 - 4x^2 + 5x$**

19. $-7, 1 - \sqrt{2}$ **$P(x) = x^3 + 5x^2 - 15x - 7$**

20. $-4, -\sqrt{7}$ **$P(x) = x^3 + 4x^2 - 7x - 28$**