5-6 Reteaching

The Fundamental Theorem of Algebra

Problem

What are all the complex roots of $x^4 + x^3 - 2x^2 + 4x - 24 = 0$?

Because this is a fourth-degree polynomial, you know it will have four roots.

Step 1 Because the polynomial is already in standard form, you can use the Rational Root Theorem to determine possible rational roots. The possible rational roots are: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$.

Step 2 Evaluate the polynomial for each possible root until you find one that causes the polynomial to equal zero. This is a rational root. In this case, one rational root is 2.

Step 3 Use synthetic division with a divisor of 2 to begin factoring the polynomial.

2	1	1	−2	4	−24
		2	6	8	24
	1	3	4	12	0

$x^3 + 3x^2 + 4x + 12 = 0$

Step 4 Repeat Steps 1–3 until you have a polynomial of degree 2 or less.

−3	1	3	4	12
		−3	0	−12
	1	0	4	0

$x^2 + 4 = 0$

Step 5 If the dividend is a second-degree polynomial, factor to find any additional roots. If the dividend does not factor easily, use the Quadratic Formula to find the additional roots.

$$\frac{-0 \pm \sqrt{0^2 - 4(1)(4)}}{2(1)} = \frac{\pm\sqrt{-16}}{2} = \frac{\pm 4i}{2} = \pm 2i$$

The four roots of $x^4 + x^3 - 4x^2 + 2x - 24 = 0$ are 2, −3, $2i$, and $-2i$.

Exercises

Find all the complex roots of each polynomial.

1. $x^4 - 8x^3 + 11x^2 + 40x - 80$
$4, \sqrt{5}, -\sqrt{5}$

2. $4x^4 - x^3 - 12x^2 + 4x - 16$
$2, -2, \frac{1 + 3i\sqrt{7}}{8}, \frac{1 - 3i\sqrt{7}}{8}$

3. $x^6 + 2x^5 + 7x^4 + 20x^3 - 21x^2 + 18x - 27$
$-3, 1, 3i, -3i, i, -i$

4. $x^3 - 4x^2 + 4x - 16$
$4, 2i, -2i$

5-6 Reteaching (continued)

The Fundamental Theorem of Algebra

Problem

What are the zeros of $f(x) = x^3 + 4x^2 - x - 10$?

The possible rational roots are $\pm 1, \pm 2, \pm 5, \pm 10$.

$$\begin{array}{r|rrrr} 1 & 1 & 4 & -1 & -10 \\ & & 1 & 5 & 4 \\ \hline & 1 & 5 & 4 & -6 \end{array} \qquad \begin{array}{r|rrrr} -1 & 1 & 4 & -3 & -10 \\ & & -1 & -3 & 6 \\ \hline & 1 & 3 & -6 & -4 \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & 4 & -1 & -10 \\ & & 2 & 12 & 22 \\ \hline & 1 & 6 & 11 & 12 \end{array} \qquad \begin{array}{r|rrrr} -2 & 1 & 4 & -1 & -10 \\ & & -2 & -4 & 10 \\ \hline & 1 & 2 & -5 & 0 \end{array}$$

Use synthetic division to test each possible rational root until you get a remainder of zero.

So -2 is one of the roots.

$x^3 + 4x^2 - x - 10 = (x + 2)(x^2 + 2x - 5)$

Use the coefficients from synthetic division to obtain the quadratic factor.

Because $x^2 + 2x - 5$ cannot be factored, use the Quadratic Formula to solve $x^2 + 2x - 5 = 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{4 - 4(1)(-5)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{24}}{2}$$

$$x = \frac{-2 \pm 2\sqrt{6}}{2}$$

$$x = -1 \pm \sqrt{6}$$

The polynomial function $f(x) = x^3 + 4x^2 - x - 10$ has one rational zero, -2, and two irrational zeros, $-1 + \sqrt{6}$ and $-1 - \sqrt{6}$.

Exercises

What are the zeros of each function?

5. $f(x) = x^3 - 2x^2 + 4x - 3$
$1, \frac{1 \pm i\sqrt{11}}{2}$

6. $f(x) = x^3 - 3x^2 - 15x + 125$
$-5, 4 \pm 3i$

7. $f(x) = 3x^3 - 2x^2 - 15x + 10$
$\frac{2}{3}, -\sqrt{5}, \sqrt{5}$

8. $f(x) = x^4 - 4x^3 + 8x^2 - 16x + 16$
$2, -2i, 2i$

9. $f(x) = x^4 - 3x^2 + 2$
$-1, 1, -\sqrt{2}, \sqrt{2}$

10. $f(x) = x^3 - 2x^2 - 17x - 6$
$-3, \frac{5 \pm \sqrt{33}}{2}$