Name ______________________ Class ____________ Date ____________

5-7 Reteaching

The Binomial Theorem

You can find the coefficients of a binomial expansion in Pascal's Triangle.

To create Pascal's Triangle, start by writing a triangle of 1's. This triangle forms the first two rows. Each row has one more element than the one above it. Each row begins with a 1, and then each element is the sum of the two closest elements in the row above. The last element in each row is a 1.

Pascal's Triangle

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

Problem

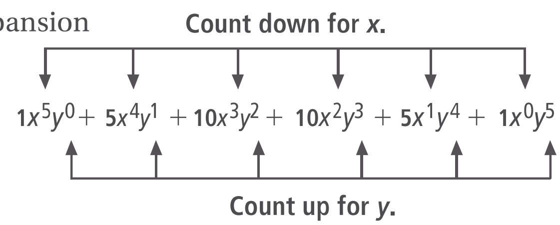
What is the expansion of $(x + y)^5$? Use Pascal's Triangle.

Step 1 The power of the binomial corresponds to the second number in each row of Pascal's Triangle. Because the power of this binomial is 5, use the row of Pascal's Triangle with 5 as the second number. The numbers of this row are the coefficients of the expansion.

Pascal's Triangle

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

Step 2 The exponents of the x-terms of the expansion begin with the power of the binomial and decrease to 0. The exponents of the y-terms of the expansion begin with 0 and increase to the power of the binomial.



Step 3 Simplify all terms to write the expansion in standard form.

$(x + y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$

Exercises

Write the expansion of each binomial.

1. $(a + b)^3$ $a^3 + 3a^2b + 3ab^2 + b^3$

2. $(x - y)^4$ $x^4 - 4x^3y + 6x^2y^2 - 4xy^3 + y^4$

3. $(r + 1)^5$ $r^5 + 5r^4 + 10r^3 + 10r^2 + 5r + 1$

4. $(a - b)^6$ $a^6 - 6a^5b + 15a^4b^2 - 20a^3b^3 + 15a^2b^4 - 6ab^5 + b^6$

5-7 Reteaching (continued)

The Binomial Theorem

- The *Binomial Theorem* states that for any binomial $(a + b)$ and any positive integer n, $(a+b)^n = {}_nC_0a^n + {}_nC_1a^{n-1}b + {}_nC_2a^{n-2}b^2 + \ldots + {}_nC_{n-1}ab^{n-1} + {}_nC_nb^n$.
- The theorem provides an effective method for expanding any power of a binomial.

Evaluate the combination ${}_nC_k$ as $\dfrac{n!}{k!(n-k)!}$.

Problem

What is the expansion of $(3x + 2)^3$? Use the Binomial Theorem.

Step 1 Determine a, b, and n.

$a = 3x,\ b = 2,\ n = 3$

Step 2 Use the formula to write the equation.

$(3x+2)^3 = {}_3C_0(3x)^3 + {}_3C_1(3x)^2(2) + {}_3C_2(3x)(2)^2 + {}_3C_3(2)^3$

Step 3 Simplify.

$= 1(27x^3) + 3(9x^2)(2) + 3(3x)(4) - 1(8)$

$= 27x^3 + 54x^2 + 36x + 8$

Exercises

Fill in the correct coefficients, variables, and exponents for the expanded form of each binomial.

5. $(x+y)^4 = x^{\square} + \square x^3y + 6x^{\square}y^2 + \square xy^{\square} + \square^4$ 4; 4; 2; 4; 3; y

6. $(z-y)^3 = z^{\square} - \square z^2y + \square zy^{\square} - \square^3$ 3; 3; 3; 2; y

7. $(x+z)^5 = x^{\square} + \square x^4z + 10x^{\square}z^2 + \square x^2z^{\square} + \square xz^4 + \square^5$ 5; 5; 3; 10; 3; 5; z

Write the expansion of each binomial. Use the Binomial Theorem.

8. $(x+y)^5$

$x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$

9. $(x-y)^5$

$x^5 - 5x^4y + 10x^3y^2 - 10x^2y^3 + 5xy^4 - y^5$

10. $(2x+y)^3$

$8x^3 + 12x^2y + 6xy^2 + y^3$

11. $(x+3y)^4$

$x^4 + 12x^3y + 54x^2y^2 + 108xy^3 + 81y^4$

12. $(x-2y)^5$ $x^5 - 10x^4y + 40x^3y^2 - 80x^2y^3 + 80xy^4 - 32y^5$

13. $(2x-y)^5$

$32x^5 - 80x^4y + 80x^3y^2 - 40x^2y^3 + 10xy^4 - y^5$

14. $(x-3y)^4$

$x^4 - 12x^3y + 54x^2y^2 - 108xy^3 + 81y^4$

15. $(4x-y)^3$

$64x^3 - 48x^2y + 12xy^2 - y^3$

16. $(x-1)^5$

$x^5 - 5x^4 + 10x^3 - 10x^2 + 5x - 1$

17. $(1-x)^3$

$1 - 3x + 3x^2 - x^3$

18. $(x^2+1)^3$ $x^6 + 3x^4 + 3x^2 + 1$

19. $(y^2+a)^4$ $y^8 + 4y^6a + 6y^4a^2 + 4y^2a^3 + a^4$