5-9 Reteaching

Transforming Polynomial Functions

Problem

What is the equation of the graph of $y = x^3$ under the following transformations?

- vertical stretch by a factor of 4
- reflection across the x-axis
- horizontal translation 2 units left
- vertical translation 3 units up

Step 1 Begin by writing the general equation for stretching, reflecting, and/or translating the cubic parent function $y = a(x - h)^3 + k$.
a = vertical stretch h = horizontal translation k = vertical translation
If a function is reflected in the x-axis, a is negative.

Step 2 The vertical stretch is 4 and the transformed function is reflected across the x-axis.
$a = -4$

Step 3 The horizontal translation is 2 units left. This is the negative x direction, so h is negative.
$h = -2$

Step 4 The vertical translation is 3 units up. This is the positive y direction, so k is positive.
$k = 3$

Step 5 Substitute a, h, and k into the general equation.
$y = a(x - h)^3 + k = -4[x - (-2)]^3 + 3 = -4(x + 2)^3 + 3$

Exercises

Determine the equation of the graph of $y = x^3$ under each set of transformations.

1. a reflection across the x-axis, a vertical translation 5 units up, and a horizontal translation 8 units right $y = -(x - 8)^3 + 5$

2. a vertical stretch by a factor of $\frac{1}{4}$, a reflection across the y-axis, and a vertical translation 2 units down $y = \frac{1}{4}(-x)^3 - 2$

3. a vertical stretch by a factor of 6, a horizontal translation 3 units left, and a vertical translation 1 unit up $y = 6(x + 3)^3 + 1$

5-9 Reteaching (continued)

Transforming Polynomial Functions

Problem

What is a quartic function with only two real zeros $x = -2$ and $x = 4$?

There are two methods you can use to find different quartic functions with only the two real zeros given.

The first method uses transformation.

Step 1 In this problem, you want zeros that are 6 units apart $(4 - (-2) = 6)$. Divide 6 in half because the basic quartic is centered on the y-axis.

Step 2 Raise this quotient to the fourth power (because you are trying to find a quartic function). This is how many units down you are translating the quartic. Because you are translating down, k will be negative. $3^4 = 81$, so $k = -81$.

Step 3 Once you vertically translate the parent quartic function down, the positive zero is located at 3 on the x-axis. The difference between 3 and 4 (the largest zero of the two given in the problem) is 1. You want to translate the quartic one unit to the right, so $h = 1$.

Step 4 Substitute the values for h and k into the general equation. One quartic function with the desired zeros is $y = (x - 1)^4 - 81$.

You can also use an algebraic method to find a quartic function with the same zeros.

Step 1 Using the Factor Theorem, substitute the zeros into the factored form of the quartic function and multiply by $Q(x)$.
$y = (x + 2)(x - 4) \cdot Q(x)$

Step 2 $Q(x)$ is any quadratic that has no real zeros. To keep things simple, use $Q(x) = x^2 + 1$. Simplify the equation.
$y = (x + 2)(x - 4)(x^2 + 1)$
A different quartic function with the desired zeros is $y = x^4 - 2x^3 - 7x^2 - 2x - 8$.

Exercises

Find a quartic function with the given x-values as its only real zeros using transformations.

4. $x = -1$ and $x = 3$ $y = (x - 1)^4 - 16$

5. $x = 5$ and $x = 7$ $y = (x - 6)^4 - 1$

Find a quartic function with the given x-values as its only real zeros using the algebraic method.

6. $x = 4$ and $x = -2$
$y = x^4 - 2x^3 - 7x^2 - 2x - 8$

7. $x = -4$ and $x = 4$
$y = x^4 - 15x^2 - 16$