Name ______________________ Class ____________ Date ____________

7-4 Reteaching

Properties of Logarithms

You can write a logarithmic expression containing more than one logarithm as a single logarithm as long as the bases are equal. You can write a logarithm that contains a number raised to a power as a logarithm with the power as a coefficient. To understand the following properties, remember that logarithms are powers.

Name	Formula	Why?
Product Property	$\log_b mn = \log_b m + \log_b n$	When you multiply two powers, you add the exponents. Example: $2^6 \cdot 2^2 = 2^{(6+2)} = 2^8$
Quotient Property	$\log_b \frac{m}{n} = \log_b m - \log_b n$	When you divide two powers, you subtract the exponents. Example: $\frac{2^6}{2^2} = 2^{(6-2)} = 2^4$
Power Property	$\log_b m^n = n\log_b m$	When you raise a power to a power, you multiply the exponents. Example: $(2^6)^2 = 2^{(6 \cdot 2)} = 2^{12}$

Problem

What is $2\log_2 6 - \log_2 9 + \frac{1}{3}\log_2 27$ written as a single logarithm?

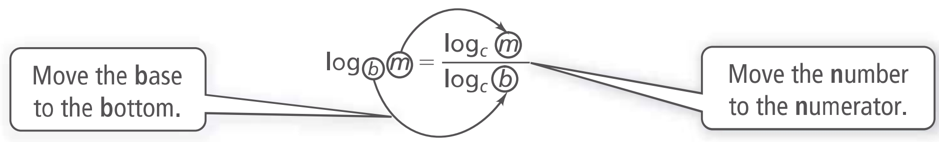
$2\log_2 6 - \log_2 9 + \frac{1}{3}\log_2 27$	$= \log_2 6^2 - \log_2 9 + \log_2 27^{\frac{1}{3}}$	Use the Power Property twice.
	$= \log_2 36 - \log_2 9 + \log_2 3$	$6^2 = 36$, $27^{\frac{1}{3}} = \sqrt[3]{27} = 3$
	$= (\log_2 36 - \log_2 9) + \log_2 3$	Group two of the logarithms. Use order of operations.
	$= \log_2 \frac{36}{9} + \log_2 3$	Quotient Property
	$= \log_2\left(\frac{36}{9} \cdot 3\right)$	Product Property
	$= \log_2 12$	Simplify.

As a single logarithm, $2\log_2 6 - \log_2 9 + \frac{1}{3}\log_2 27 = \log_2 12$.

7-4 Reteaching (continued)

Properties of Logarithms

To evaluate logarithms with any base, you can rewrite the logarithm as a quotient of two logarithms with the same base.



Problem

What is $\log_4 8$ written as a quotient of two logarithms with base 2? Simplify your answer, if possible.

$\log_4 8 = \dfrac{\log_2 8}{\log_2 4}$ The base is 4 and the number is 8. Move the base to the bottom and the number to the numerator.

$= \dfrac{3}{2}$ Evaluate the logarithms in the numerator and the denominator.

Exercises

Write each logarithmic expression as a single logarithm.

1. $\log_3 13 + \log_3 3$ $\log_3 39$

2. $2 \log x + \log 5$ $\log 5x^2$

3. $\log_4 2 - \log_4 6$ $\log_4 \frac{1}{3}$

4. $3 \log_3 3 - \log_3 3$ $\log_3 9$, or 2

5. $\log_5 8 + \log_5 x$ $\log_5 8x$

6. $\log 2 - 2 \log x$ $\log \frac{2}{x^2}$

7. $\log_2 x + \log_2 y$ $\log_2 xy$

8. $3 \log_7 x - 5 \log_7 y$ $\log_7 \frac{x^3}{y^5}$

9. $4 \log x + 3 \log x$ $\log x^7$

10. $\log_5 x + 3 \log_5 y$ $\log_5 xy^3$

11. $3 \log_2 x - \log_2 y$ $\log_2 \frac{x^3}{y}$

12. $\log_2 16 - \log_2 8$ $\log_2 2$, or 1

Write each logarithm as a quotient of two common logarithms. Simplify your answer, if possible. (*Hint:* Common logarithms are logarithms with base 10.)

13. $\log_4 12$ $\frac{\log 12}{\log 4}$

14. $\log_2 1000$ $\frac{3}{\log 2}$

15. $\log_5 16$ $\frac{\log 16}{\log 5}$

16. $\log_{11} 205$ $\frac{\log 205}{\log 11}$

17. $\log_9 32$ $\frac{\log 32}{\log 9}$

18. $\log_{100} 51$ $\frac{\log 51}{2}$