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Reteaching

Natural Logarithms

The **natural logarithmic function** is a logarithm with base e , an irrational number.

You can write the natural logarithmic function as $y = \log_e x$, but you usually write it as $y = \ln x$.

$y = e^x$ and $y = \ln x$ are inverses, so if $y = e^x$, then $x = \ln y$.

To solve a natural logarithm equation:

- If the term containing the variable is an exponential expression, rewrite the equation in logarithmic form.
- If the term containing the variable is a logarithmic expression, rewrite the equation in exponential form.

Problem

What is the solution of $4e^{2x} - 2 = 3$?

Step 1 Isolate the term containing the variable on one side of the equation.

$$4e^{2x} - 2 = 3$$

$$4e^{2x} = 5 \quad \text{Add 2 to each side of the equation.}$$

$$e^{2x} = \frac{5}{4} \quad \text{Divide each side of the equation by 4.}$$

Step 2 Take the natural logarithm of each side of the equation.

$$\ln(e^{2x}) = \ln\left(\frac{5}{4}\right)$$

$$2x = \ln\left(\frac{5}{4}\right) \quad \text{Definition of natural logarithm}$$

Step 3 Solve for the variable.

$$x = \frac{\ln\left(\frac{5}{4}\right)}{2}$$

Divide each side of the equation by 2.

$$x \approx 0.112 \quad \text{Use a calculator.}$$

Step 4 Check the solution.

$$4e^{2(0.112)} - 2 \stackrel{?}{=} 3$$

$$4e^{0.224} - 2 \stackrel{?}{=} 3$$

$$3.004 \approx 3$$

The solution is $x \approx 0.112$.

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Reteaching (continued)

Natural Logarithms

Problem

What is the solution of $\ln(t - 2)^2 + 1 = 6$? Round your answer to the nearest thousandth.

Step 1 Isolate the term containing the variable on one side of the equation.

$$\ln(t - 2)^2 + 1 = 6$$

$$\ln(t - 2)^2 = 5 \quad \text{Subtract 1 from each side of the equation.}$$

Step 2 Raise each side of the equation to the base e .

$$e^{\ln(t-2)^2} = e^5$$

$$(t - 2)^2 = e^5 \quad \text{Definition of natural logarithm}$$

Step 3 Solve for the variable.

$$t - 2 = \pm e^{\frac{5}{2}} \quad \text{Take the square root of each side of the equation.}$$

$$t = 2 \pm e^{\frac{5}{2}} \quad \text{Add 2 to each side of the equation.}$$

$$t \approx 14.182 \text{ or } -10.182 \quad \text{Use a calculator.}$$

Step 4 Check the solution.

$$\ln(14.182 - 2)^2 \stackrel{?}{=} 5 \quad \ln(-10.182 - 2)^2 \stackrel{?}{=} 5$$

$$4.9999 \approx 5 \quad 4.9999 \approx 5$$

The solutions are $t \approx 14.182$ and -10.182 .

Exercises

Use natural logarithms to solve each equation. Round your answer to the nearest thousandth. Check your answers.

1. $2e^x = 4$ **0.693**

2. $e^{4x} = 25$ **0.805**

3. $e^x = 72$ **4.277**

4. $e^{3x} = 124$ **1.607**

5. $12e^{3x-2} = 8$ **0.532**

6. $\frac{1}{2}e^{6x} = 5$ **0.384**

Solve each equation. Round your answer to the nearest thousandth. Check your answers.

7. $\ln(x - 3) = 2$ **10.389**

8. $\ln 2t = 4$ **27.299**

9. $1 + \ln x^2 = 2$ **± 1.649**

10. $\ln(2x - 5) = 3$ **12.543**

11. $\frac{1}{3}\ln 2t = 1$ **10.043**

12. $\ln(t - 4)^2 + 2 = 5$
8.482, -0.482