Chapter 9 Sequences and Series

A **sequence** is an ordered list of numbers called **terms**.

A **recursive definition** gives the first term and defines the other terms by relating each term after the first term to the one before it.

An **explicit formula** expresses the nth term in a sequence in terms of n, where n is a positive integer.

Arithmetic Sequence

A recursive definition for an arithmetic sequence with a starting value a and a common difference d has two parts:
$a_1 = a$: initial condition
$a_{n+1} = a_n + d$, for $n \geq 1$: recursive formula
An explicit definition for this sequence is the formula:
$a_n = a + (n - 1)d$ for $n \geq 1$.

The **arithmetic mean** of two numbers x and y is the average of the two numbers $\frac{x + y}{2}$.

Geometric Sequence

A recursive definition for a geometric sequence with a starting value a and a common ratio r has two parts:
$a_1 = a$: initial condition
$a_{n+1} = a_n \cdot r$, for $n \geq 1$: recursive formula
An explicit definition for this sequence is the formula:
$a_n = ar^{n-1}$, for $n \geq 1$.

The geometric mean of two positive numbers x and y is $\sqrt{xy}$.

A **series** is the expression for the sum of the terms of a sequence.

Sum of a Finite Arithmetic Series

The sum S_n of a finite arithmetic series

$a_1 + a_2 + a_3 + \cdots + a_n$ is $S_n = \frac{n}{2}(a_1 + a_n)$

where a_1 is the first term, a_n is the nth term, and n is the number of terms.

Sum of a Finite Geometric Series

The sum S_n of a finite geometric series

$a_1 + a_1r + a_1r^2 + \cdots + a_1r^{n-1}$ is $S_n = \frac{a_1(1 - r^n)}{1 - r}$

where a_1 is the first term, r is the common ratio, and n is the number of terms.

Sum of an Infinite Geometric Series

An infinite geometric series with $|r| < 1$ converges to the sum S given by the following formula:

$S = \frac{a_1}{1 - r}$.

In a geometric series, when $|r| < 1$, the series converges to $S = \frac{a_1}{1 - r}$. When $|r| \geq 1$, the series diverges.