Chapter 11 Probability and Statistics

Fundamental Counting Principle

If event M can occur in m ways and is followed by event N that can occur in n ways, then event M followed by event N can occur in $m \cdot n$ ways.

Number of Permutations

The number of permutations of n items of a set arranged r items at a time is

$${}_nP_r = \frac{n!}{(n-r)!} \text{ for } 0 \leq r \leq n.$$

Number of Combinations

The number of combinations of n items of a set chosen r items at a time is

$${}_nC_r = \frac{n!}{r!(n-r)!} \text{ for } 0 \leq r \leq n.$$

Probability of *A* and *B*

If A and B are independent events, then

$P(A \text{ and } B) = P(A) \cdot P(B)$.

Probability of *A* or *B*

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

If A and B are mutually exclusive events, then

$P(A \text{ or } B) = P(A) + P(B)$.

Conditional Probability

For any two events A and B with $P(A) \neq 0$, the probability of event B, given event A, is:

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

Binomial Probability

For repeated independent trials, each with a probability of success p and a probability of failure q (with $p + q = 1$), the probability of x successes in n trials is $P(x) = {}_nC_x p^x q^{n-x}$.

Binomial Theorem Using Combinations

For every positive integer n, use the combinations formula ${}_nC_r$ to expand $(a + b)^n$:

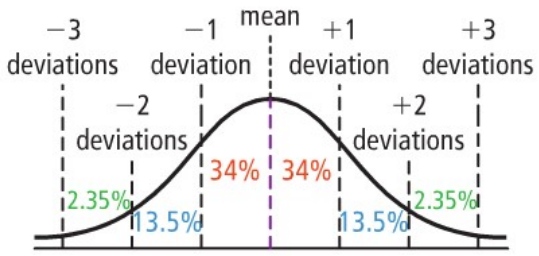
$$(a + b)^n = {}_nC_0 a^n + {}_nC_1 a^{n-1} b + {}_nC_2 a^{n-2} b^2 + \cdots + {}_nC_{n-1} a b^{n-1} + {}_nC_n b^n$$

Mean, Variance, and Standard Deviation

Mean: $\bar{x} = \dfrac{x_1 + x_2 + x_3 + \cdots + x_n}{n}$

Variance: $\sigma^2 = \dfrac{\Sigma(x - \bar{x})^2}{n}$

Standard deviation: $\sigma = \sqrt{\dfrac{\Sigma(x - \bar{x})^2}{n}}$



A normal distribution has a symmetric bell shape centered on the mean.

In a normal distribution,

- 68% of data fall within one standard deviation of the mean
- 95% of data fall within two standard deviations of the mean
- 99.7% of data fall within three standard deviations of the mean

A **sample** is part of a **population**. For a **random sample**, all members of the population are equally likely to be chosen. A **bias** is a systematic error introduced by the sampling method.