Chapter 13 Periodic Functions and Trigonometry

A **periodic function** repeats a pattern of y-values at regular intervals. One complete pattern is called a **cycle**. A cycle may begin at any point on the graph. The **period** of a function is the length of one cycle. The **amplitude** of a periodic function is half the difference between its maximum and minimum values.

Convert Between Radians and Degrees

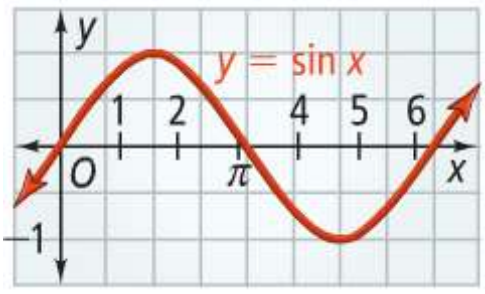
Use the proportion $\frac{d^\circ}{180^\circ} = \frac{r \text{ radians}}{\pi \text{ radians}}$ to convert between radians and degrees.

To convert degrees to radians, multiply by $\frac{\pi \text{ radians}}{180^\circ}$.

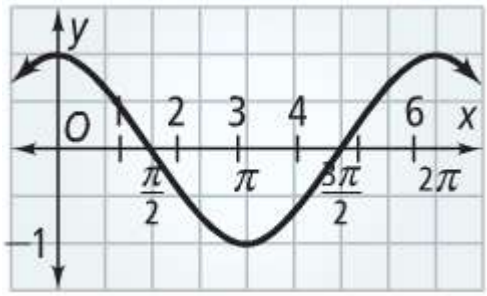
To convert radians to degrees, multiply by $\frac{180^\circ}{\pi \text{ radians}}$.

Length of an Intercepted Arc

For a circle of radius r and a central angle of measure θ (in radians), the length s of the intercepted arc is $s = r\theta$.



sine



cosine

Sine and Cosine Functions

	Sine	Cosine
Parents	$y = \sin x$	$y = \cos x$
Reflection across x-axis	$y = -\sin x$	$y = -\cos x$
Amplitude $\lvert a \rvert$	$y = a \sin x$	$y = a \cos x$
Period $\frac{2\pi}{b}$, $b > 0$	$y = \sin bx$	$y = \cos bx$
Translation horizontal by h vertical by k	$y = \sin (x - h) + k$	$y = \cos (x - h) + k$

Tangent Function

Parent	$y = \tan x$
Reflection across x-axis	$y = -\tan x$
Period $\frac{\pi}{b}$	$y = \tan bx$
Translation horizontal by h vertical by k	$y = \tan (x - h) + k$
Asymptotes ($\tan bx$)	$x = n\frac{\pi}{2b}$, n odd

Basic Identities

Reciprocal Identities:

$\csc \theta = \frac{1}{\sin \theta}$ $\sec \theta = \frac{1}{\cos \theta}$ $\tan \theta = \frac{1}{\cot \theta}$

$\sin \theta = \frac{1}{\csc \theta}$ $\cos \theta = \frac{1}{\sin \theta}$ $\cot \theta = \frac{1}{\tan \theta}$

Tangent Identity: $\tan \theta = \frac{\sin \theta}{\cos \theta}$

Cotangent Identity: $\cot \theta = \frac{\cos \theta}{\sin \theta}$

Pythagorean Identities

$\cos^2 \theta + \sin^2 \theta = 1$ $1 + \tan^2 \theta = \sec^2 \theta$ $\cot^2 \theta + 1 = \csc^2 \theta$