Name ________________________________ Class ____________ Date ____________

8-3 Reteaching

Rational Functions and Their Graphs

A rational function may have one or more types of discontinuities: holes (removable points of discontinuity), vertical asymptotes (non-removable points of discontinuity), or a horizontal asymptote.

If	Then	Example
a is a zero with multiplicity m in the numerator and multiplicity n in the denominator, and $m \geq n$	hole at $x = a$	$f(x) = \frac{(x - 5)(x + 6)}{(x - 5)}$ hole at $x = 5$
a is a zero of the denominator only, or a is a zero with multiplicity m in the numerator and multiplicity n in the denominator, and $m < n$	vertical asymptote at $x = a$	$f(x) = \frac{x^2}{x - 3}$ vertical asymptote at $x = 3$

Let p = degree of numerator.
Let q = degree of denominator.

• $m < n$	horizontal asymptote at $y = 0$	$f(x) = \frac{4x^2}{7x^2 + 2}$ horizontal asymptote at $y = \frac{4}{7}$
• $m > n$	no horizontal asymptote exists	
• $m = n$	horizontal asymptote at $y = \frac{a}{b}$, where a and b are coefficients of highest degree terms in numerator and denominator	

Problem

What are the points of discontinuity of $y = \frac{x^2 + x - 6}{3x^2 - 12}$, if any?

Step 1 Factor the numerator and denominator completely. $y = \frac{(x - 2)(x + 3)}{3(x - 2)(x + 2)}$

Step 2 Look for values that are zeros of both the numerator and the denominator. The function has a hole at $x = 2$.

Step 3 Look for values that are zeros of the denominator only. The function has a vertical asymptote at $x = -2$.

Step 4 Compare the degrees of the numerator and denominator. They have the same degree. The function has a horizontal asymptote at $y = \frac{1}{3}$.

Exercises

Find the vertical asymptotes, holes, and horizontal asymptote for the graph of each rational function.

1. $y = \frac{x}{x^2 - 9}$ **vertical asymptotes: $x = 3, x = -3$; horizontal asymptote: $y = 0$**

2. $y = \frac{6x^2 - 6}{x - 1}$ **hole: $x = 1$**

3. $y = \frac{4x + 5}{3x + 2}$ **vertical asymptote: $x = -\frac{2}{3}$; horizontal asymptote: $y = \frac{4}{3}$**

8-3 Reteaching (continued)

Rational Functions and Their Graphs

Before you try to sketch the graph of a rational function, get an idea of its general shape by identifying the graph's holes, asymptotes, and intercepts.

Problem

What is the graph of the rational function $y = \frac{x + 3}{x + 1}$?

Step 1 Identify any holes or asymptotes.

no holes; vertical asymptote at $x = -1$; horizontal asymptote at $y = \frac{1}{1} = 1$

Step 2 Identify any x- and y-intercepts.

x-intercepts occur when $y = 0$. y-intercepts occur when $x = 0$.

$\frac{x + 3}{x + 1} = 0$

$x + 3 = 0$

$x = -3$

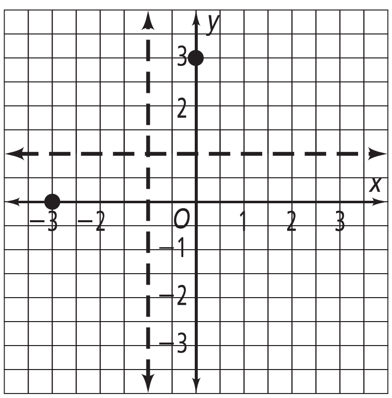
x-intercept at -3

$y = \frac{0 + 3}{0 + 1}$

$y = 3$

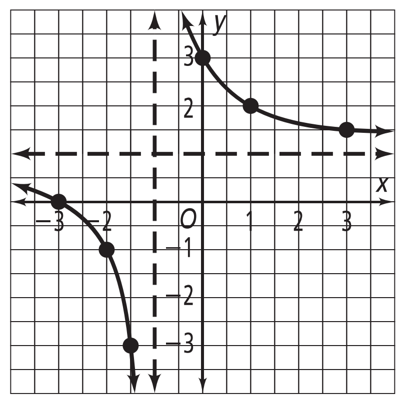
y-intercept at 3

Step 3 Sketch the asymptotes and intercepts.



Step 4 Make a table of values, plot the points, and sketch the graph.

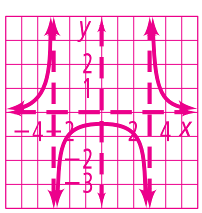
x	y
−2	−1
−1.5	−3
1	2
3	1.5



Exercises

Graph each function. Include the asymptotes.

4. $y = \frac{4}{x^2 - 9}$



5. $y = \frac{x^2 + 2x - 2}{x - 1}$