Name _____ Class _____ Date _____

8-6 Reteaching

Solving Rational Equations

When one or both sides of a rational equation has a sum or difference, multiply each side of the equation by the LCD to eliminate the fractions.

Problem

What is the solution of the rational equation $\frac{6}{x} + \frac{x}{2} = 4$? Check the solutions.

$2x\left(\frac{6}{x}\right) + 2x\left(\frac{x}{2}\right) = 2x(4)$	Multiply each term on both sides by the LCD, $2x$.
$2\cancel{x}\left(\frac{6}{\cancel{x}}\right) + \cancel{2}x\left(\frac{x}{\cancel{2}}\right) = 2x(4)$	Divide out the common factors.
$12 + x^2 = 8x$	Simplify.
$x^2 - 8x + 12 = 0$	Write the equation in standard form.
$(x - 2)(x - 6) = 0$	Factor.
$x - 2 = 0$ or $x - 6 = 0$	Use the Zero-Product Property.
$x = 2$ or $x = 6$	Solve for x.

Check

$\frac{6}{x} + \frac{x}{2} \stackrel{?}{=} 4$ $\quad$ $\frac{6}{x} + \frac{x}{2} \stackrel{?}{=} 4$

$\frac{6}{2} + \frac{2}{2} \stackrel{?}{=} 4$ $\quad$ $\frac{6}{6} + \frac{6}{2} \stackrel{?}{=} 4$

$3 + 1 \stackrel{?}{=} 4$ $\quad$ $1 + 3 \stackrel{?}{=} 4$

$4 = 4$ ✓ $\quad$ $4 = 4$ ✓

The solutions are $x = 2$ and $x = 6$.

Exercises

Solve each equation. Check the solutions.

1. $\frac{10}{x + 3} + \frac{10}{3} = 6$ $\frac{3}{4}$

2. $\frac{1}{x - 3} = \frac{x - 4}{x^2 - 27}$ $\frac{39}{7}$

3. $\frac{6}{x - 1} + \frac{2x}{x - 2} = 2$ $\frac{8}{5}$

4. $\frac{7}{3x - 12} - \frac{1}{x - 4} = \frac{2}{3}$ 6

5. $\frac{2x}{5} = \frac{x^2 - 5x}{5x}$ −5

6. $\frac{8(x - 1)}{x^2 - 4} = \frac{4}{x - 2}$ 4

7. $x + \frac{4}{x} = \frac{25}{6}$ $\frac{3}{2}, \frac{8}{3}$

8. $\frac{2}{x} + \frac{6}{x - 1} = \frac{6}{x^2 - x}$ no solution

9. $\frac{2}{x} + \frac{1}{x} = 3$ 1

10. $\frac{4}{x - 1} = \frac{5}{x - 1} + 2$ $\frac{1}{2}$

11. $\frac{1}{x} = \frac{5}{2x} + 3$ $-\frac{1}{2}$

12. $\frac{x + 6}{5} = \frac{2x - 4}{5} - 3$ 25

Name _____ Class _____ Date _____

8-6 Reteaching (continued)

Solving Rational Equations

You often can use rational equations to model and solve problems involving rates.

Problem

Quinn can refinish hardwood floors four times as fast as his apprentice, Jack. They are refinishing 100 ft^2 of flooring. Working together, Quinn and Jack can finish the job in 3 h. How long would it take each of them working alone to refinish the floor?

Let x be Jack's work rate in ft^2/h. Quinn's work rate is four times faster, or $4x$.

square feet refinished per hour by Jack and Quinn together	=	square feet of floor they refinish together	÷	hours worked together
ft^2/h	=	ft^2	÷	h

$x + 4x = \frac{100}{3}$ Their work rates sum to 100 ft^2 in 3 h.

$3(x) + 3(4x) = 3\left(\frac{100}{3}\right)$ They work for 3 h. Refinished floor area = rate × time.

$15x = 100$ Simplify.

$x \approx 6.67$ Divide each side by 15.

Jack works at the rate of 6.67 ft^2/h. Quinn works at the rate of 26.67 ft^2/h.

Let j be the number of hours Jack takes to refinish the floor alone, and let q be the number of hours Quinn takes to refinish the floor alone.

$$6.67 = \frac{100}{j} \qquad 26.67 = \frac{100}{q}$$

$$j(6.67) = j\left(\frac{100}{j}\right) \qquad q(26.67) = q\left(\frac{100}{q}\right)$$

$$6.67j = 100 \qquad 26.67q = 100$$

$$j \approx 15 \qquad q \approx 3.75$$

Jack would take 15 h and Quinn would take 3.75 h to refinish the floor alone.

Exercises

13. An airplane flies from its home airport to a city and back in 5 h flying time. The plane travels the 720 mi to the city at 295 mi/h with no wind. How strong is the wind on the return flight? Is the wind a headwind or a tailwind? **about 14 mi/h; headwind**

14. Miguel can complete the decorations for a school dance in 5 days working alone. Nasim can do it alone in 3 days, and Denise can do it alone in 4 days. How long would it take the three students working together to decorate? **about 1.3 days**