Name ______________________ Class ____________ Date ____________

9-5 Reteaching

Geometric Series

- The **sum of a finite geometric series** is $S_n = \frac{a_1(1 - r^n)}{1 - r}$, where a_1 is the first term, r is the common ratio, and n is the number of terms.
- The **sum of an infinite geometric series** with $|r| < 1$ is $S = \frac{a_1}{1 - r}$, where a_1 is the first term and r is the common ratio. If $|r| \geq 1$, then the series has no sum.

Problem

What is the sum of the first ten terms of the geometric series
$8 + 16 + 32 + 64 + 128 + \ldots$?

$a_1 = 8$	a_1 is the first term in the series.
$r = \frac{16}{8} = \frac{32}{16} = \frac{64}{32} = \frac{128}{64} = 2$	Simplify the ratio formed by any two consecutive terms to find r.
$n = 10$	n is the number of terms in the series to be added together.
$S_{10} = \frac{8(1 - 2^{10})}{1 - 2}$	Substitute $a_1 = 8$, $r = 2$, and $n = 10$ into the formula for the sum of a finite geometric series.
$= \frac{8(-1023)}{-1}$	Simplify inside the parentheses.
$= 8184$	Simplify.

Exercises

Evaluate the finite series for the specified number of terms.

1. $3 + 12 + 48 + 192 + \ldots; n = 6$ **4095**

2. $8 + 2 + \frac{1}{2} + \frac{1}{8} + \ldots; n = 5$ $\frac{341}{32}$

3. $-10 - 5 - 2.5 - 1.25 - \ldots; n = 7$ $-\frac{635}{32}$

4. $10 + (-5) + \frac{5}{2} + \left(-\frac{5}{4}\right) + \ldots; n = 11$ $\frac{3415}{512}$

Evaluate each infinite geometric series.

5. $10 + 5 + 2.5 + \ldots$ **20**

6. $-1 + \frac{2}{11} - \frac{4}{121} + \ldots$ $-\frac{11}{13}$

7. $\frac{1}{4} + \frac{7}{32} + \frac{49}{256} + \ldots$ **2**

8. $\frac{1}{2} - \frac{1}{5} + \frac{2}{25} - \ldots$ $\frac{5}{14}$

9. $-\frac{1}{6} + \frac{1}{12} - \frac{1}{24} + \ldots$ $-\frac{1}{9}$

10. $20 + 16 + \frac{64}{5} + \ldots$ **100**

11. $12 + 4 + \frac{4}{3} + \ldots$ **18**

12. $\frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \ldots$ $\frac{1}{6}$

13. $\frac{2}{3} + \frac{2}{15} + \frac{2}{75} + \ldots$ $\frac{5}{6}$

9-5 Reteaching (continued)

Geometric Series

Problem

Your neighbor hosts a family reunion every year. In 2000, it costs $1500 to host the reunion. Their expenses have decreased by 10% per year by asking family members to contribute food and party supplies.

a. What is a rule for the cost of the family reunion?
b. What was the cost of the reunion in 2005?
c. What was the total cost for hosting the family reunions from 2000 to 2009?

The cost is a geometric sequence that decreases by the same percent each year.

$a_n = ar^{n-1}$ Write the explicit formula.

$a_n = (1500)(0.90)^{n-1}$ Substitute $a_1 = 1500$, $r = 1 - 0.10 = 0.90$ in the explicit formula.

To find the cost of the reunion in 2005 ($n = 6$), substitute values into the explicit formula.

$a_n = ar^{n-1}$ Write the explicit formula.

$a_n = (1500)(0.90)^{6-1}$ Substitute $a_1 = 1500$, $r = 0.90$, $n = 6$ in the formula.

$a_n \approx 886$ Simplify.

The cost of hosting the reunion in 2005 was $886.

To find the total of hosting the reunions from 2000 to 2009, $\sum_{n=1}^{10}(1500)(0.90)^{n-1}$, find the sum of the geometric series.

$S_n = \dfrac{a_1(1 - r^n)}{1 - r}$ Write the formula for the sum of a geometric series.

$S_{10} = \dfrac{1500(1 - 0.90^{10})}{1 - 0.90}$ Substitute $a_1 = 1500$, $r = 0.90$, $n = 10$ in the formula.

$S_{10} \approx 9770$ Simplify.

The cost of hosting the reunions from 2000 to 2009 was $9770.

Exercise

14. In 1990, a vacation package cost $400. The cost has increased 10% per year.

a. What are the values of a_1 and r? $a_1 = 400, r = 1.1$
b. What is a rule for the cost of the vacation? $a_n = (400)(1.10)^{n-1}$
c. What was cost of the vacation in 1995? **$644.20**
d. What was the total cost of the vacations from 1990 to 1999? **$6374.97**
e. If the pattern continued until 2009, what was the total cost of the vacations? **$22,910**