13-2 Reteaching

Angles and the Unit Circle

A measurement of an angle in standard position is the measurement of the *rotation* from the initial side of the angle to the terminal side of the angle. Coterminal angles have the same terminal side.
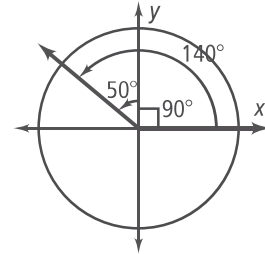
Problem

What are two angles that are coterminal with a 140° angle?

Step 1 Sketch a 140° angle in standard position.
The rotation from the initial side of the angle to the positive *y*-axis is 90°. So, the rotation from the positive *y*-axis to the terminal side of the angle is 50°. (140 − 90 = 50)

Step 2 Put your finger on the point where the initial side intersects the unit circle. Trace one rotation *counterclockwise* around the circle. Count the degrees of rotation (90°, 180°, 270°, 360°) as you pass each axis. Keep tracing to the positive *y*-axis again. The degree of rotation is now 450°. (360 + 90 = 450)
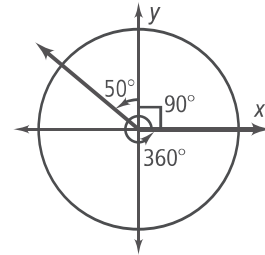
Continue tracing to the terminal side of the angle. Now the degree of rotation is 500°. (450 + 50 = 500)
A 500° angle is coterminal with a 140° angle.

Step 3 Put your finger on the point where the initial side intersects the unit circle. Trace the circle *clockwise*, counting the *negative* degrees of rotation as you pass each axis. (−90°, −180°)

Keep tracing until you reach the terminal side of the angle. The rotation from the negative *x*-axis to the terminal side of the angle is −40°. (140 − 180 = −40)
So, the total rotation is −220°. (−180 + (−40) = −220)
A −220° angle is coterminal with a 140° angle.
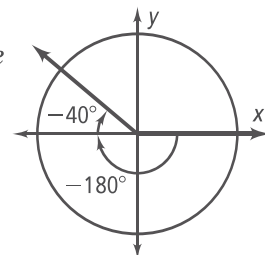
Exercises

Give one positive angle and one negative angle coterminal with the given angle. **Answers may vary. Samples are given.**

1. 20° **380°, −340°** **2.** 265° **625°, −95°** **3.** 305° **665°, −55°**

13-2 Reteaching (continued)

Angles and the Unit Circle

The coordinates of the point where the terminal side of the angle intersects the unit circle are the cosine and sine of the angle.

Problem

What are the coordinates of the point where the terminal side of a 315° angle intersects the unit circle?

Step 1 Use a compass to draw a unit circle. Use a protractor to sketch the angle. Have the terminal side of the angle intersect the circle.

Step 2 Because the terminal side is in the fourth quadrant, x is positive and y is negative.

Step 3 Use a ruler to draw the horizontal leg of the right triangle. The hypotenuse lies on the terminal side of the angle. The other leg lies on the negative y-axis.

Step 4 Because $360 - 315 = 45$, you can label the acute angles of the triangle as 45°. Use properties of special right triangles. The length of the hypotenuse is $\sqrt{2}$ times the length of a leg. Label each leg s.
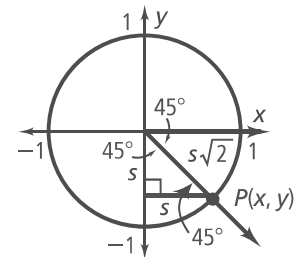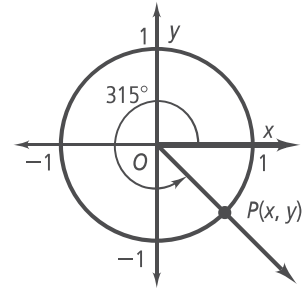
hypotenuse $= 1$

$$s\sqrt{2} = 1$$

$$s = \frac{1}{\sqrt{2}}$$

$$s = \frac{\sqrt{2}}{2}$$

each leg $= \frac{\sqrt{2}}{2}$

Step 5 The unit circle has a radius of 1 unit.

Substitute $s\sqrt{2}$ for the length of the hypotenuse.

Divide both sides by $\sqrt{2}$.

Rationalize the denominator by multiplying the fraction by $\frac{\sqrt{2}}{\sqrt{2}}$.

The coordinates of the point of intersection are $\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$.

Exercises

Find the coordinates of the point where the terminal side of each angle intersects the unit circle. These are the cosine and sine of the angle.

4. $-150°$ **5.** $30°$ **6.** $-330°$ **7.** $-45°$ **8.** $120°$ **9.** $225°$

$\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ $\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$ $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ $\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$