Name ______________________________ Class ____________ Date ____________

14-2 Reteaching

Solving Trigonometric Equations Using Inverses

You have already learned how to evaluate trigonometric functions for a given angle. You can also do the reverse. That is, you can find an angle that corresponds to a given value of a trigonometric function using your calculator. Make sure your calculator is set to radians.

The inverse sine of a is an angle θ, written $\sin^{-1} a = \theta$.
The inverse cosine of a is an angle θ, written $\cos^{-1} a = \theta$.
The inverse tangent of a is an angle θ, written $\tan^{-1} a = \theta$.

Problem

What are the radian measures of all angles whose sine is 0.93?

$\sin^{-1} a = \theta \rightarrow \sin^{-1}(0.93) = \theta$

$\sin^{-1}(0.93) \approx 1.19$

Use the inverse sine button on your calculator with $a = 0.93$. Press ENTER.

The angle measuring 1.19 radians is in Quadrant I. The sine function is positive in Quadrants I and II. In order to find all of the angles whose sine is 0.93, you must also find the angle in Quadrant II. So, $\pi - 1.19 \approx 1.95$ is another angle whose sine is 0.93 and is located in Quadrant II.

Because there are no domain restrictions, there are actually an infinite number of angles with sine 0.93. The sine function has a period of 2π, so you can find all possible angles by adding positive and negative multiples of 2π to the angles you found in Quadrants I and II.

The angles are $1.19 + 2\pi n$ and $1.95 + 2\pi n$.

Exercises

Use a calculator and inverse functions to find the radian measures of all angles having the given trigonometric values.

1. angles whose sine is 0.2
$0.20 + 2\pi n$ and $2.94 + 2\pi n$

2. angles whose sine is -0.98
$4.51 + 2\pi n$ and $4.91 + 2\pi n$

3. angles whose cosine is -0.82
$2.53 + 2\pi n$ and $3.75 + 2\pi n$

4. angles whose tangent is 3.2
$1.27 + \pi n$

5. angles whose tangent is -6.5
$1.72 + \pi n$

6. angles whose cosine is 0.4
$1.16 + 2\pi n$ and $5.12 + 2\pi n$

14-2 Reteaching (continued)

Solving Trigonometric Equations Using Inverses

You learned to find radian measures of trigonometric identities. By contrast, solutions of trigonometric equations are true only for certain values of the variable. You can still use your calculator to find the value of the trigonometric function, but you should only give solutions that are in the given domain.

Problem

What are all values for θ that satisfy the equation $4\sin\theta - \sqrt{3} = 2\sin\theta$ for $0 \leq \theta < 2\pi$?

$4\sin\theta - \sqrt{3} = 2\sin\theta$

$2\sin\theta - \sqrt{3} = 0$ Subtract $2\sin\theta$ from each side.

$2\sin\theta = \sqrt{3}$ Add $\sqrt{3}$ to each side.

$\sin\theta = \frac{\sqrt{3}}{2}$ Divide each side by 2.

$\sin^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{3}$ Use the inverse function to find one value of θ.

The sine function is also positive in Quadrant II. So another value of θ is $\pi - \frac{\pi}{3} = \frac{2\pi}{3}$.

The two solutions between 0 and 2π are $\frac{\pi}{3}$ and $\frac{2\pi}{3}$.

Exercises

Solve each equation for θ with $0 \leq \theta < 2\pi$.

7. $\sin\theta + 2\sin\theta\cos\theta = 0$
$0, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}$

8. $2\sin\theta - 4 = -2\sin\theta$
$\frac{\pi}{2}$

9. $2\cos^2\theta + \cos\theta - 1 = 0$
$\frac{\pi}{3}, \frac{5\pi}{3}, \pi$

10. $\cos\theta - 2\sin\theta\cos\theta = 0$
$\frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$

11. $\sqrt{3} + 5\sin\theta = 3\sin\theta$
$\frac{5\pi}{3}, \frac{4\pi}{3}$

12. $3\sin\theta = 1$
0.34, 2.80

13. $2\tan\theta - 4 = 0$
1.11, 4.25

14. $4\sin^2\theta - 1 = 0$
$\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

15. $2\sin^2\theta + 3\sin\theta = -1$
$\frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$

16. $\tan\theta(\sin\theta - 1) = 0$
$0, \frac{\pi}{2}, \pi$

17. $3\tan\theta = -\sqrt{3}$
$\frac{5\pi}{6}, \frac{11\pi}{6}$

18. $-5\cos\theta = \cos\theta - 3\sqrt{3}$
$\frac{\pi}{6}, \frac{11\pi}{6}$