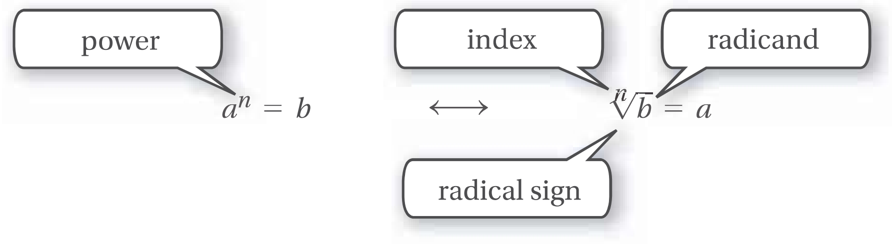
Name ______________________ Class ____________ Date ____________

6-1 Reteaching

Roots and Radical Expressions

For any real numbers a and b and any positive integer n, if a raised to the nth power equals b, then a is an nth root of b. Use the radical sign to write a root. The following expressions are equivalent:



Problem

What are the real-number roots of each radical expression?

a. $\sqrt[3]{343}$

Because $(7)^3 = 343$, 7 is a third (cube) root of 343.
Therefore, $\sqrt[3]{343} = 7$.
(Notice that $(-7)^3 = -343$, so -7 is not a cube root of 343.)

b. $\sqrt[4]{\frac{1}{625}}$

Because $\left(\frac{1}{5}\right)^4 = \frac{1}{625}$ and $\left(-\frac{1}{5}\right)^4 = \frac{1}{625}$, both $\frac{1}{5}$ and $-\frac{1}{5}$ are real-number fourth roots of $\frac{1}{625}$.

c. $\sqrt[3]{-0.064}$

Because $(-0.4)^3 = -0.064$, -0.4 is a cube root of -0.064 and is, in fact, the only one.
So, $\sqrt[3]{-0.064} = -0.4$.

d. $\sqrt{-25}$

Because $(5)^2 = (-5)^2 = 25$, neither 5 nor -5 are second (square) roots of -25. There are no real-number square roots of -25.

Exercises

Find the real-number roots of each radical expression.

1. $\sqrt{169}$ **−13, 13**

2. $\sqrt[3]{729}$ **9**

3. $\sqrt[4]{0.0016}$ **−0.2, 0.2**

4. $\sqrt[3]{-\frac{1}{8}}$ $-\frac{1}{2}$

5. $\sqrt{\frac{4}{121}}$ $-\frac{2}{11}, \frac{2}{11}$

6. $\sqrt[3]{\frac{125}{216}}$ $\frac{5}{6}$

7. $\sqrt{-\frac{4}{25}}$ **no real sq root**

8. $\sqrt[4]{0.1296}$ **−0.6, 0.6**

9. $\sqrt[3]{-0.343}$ **−0.7**

10. $\sqrt[4]{-0.0001}$ **no real 4th root**

11. $\sqrt[5]{\frac{1}{243}}$ $\frac{1}{3}$

12. $\sqrt[3]{\frac{8}{125}}$ $\frac{2}{5}$

6-1 Reteaching (continued)

Roots and Radical Expressions

You cannot assume that $\sqrt[n]{a^n} = a$. For example, $\sqrt{(-6)^2} = \sqrt{36} = 6$, not -6. This leads to the following property for any real number a:

If n is *odd* $\quad \sqrt[n]{a^n} = a$

If n is *even* $\quad \sqrt[n]{a^n} = |a|$

Problem

What is the simplified form of each radical expression?

a. $\sqrt[3]{1000x^3y^9}$

$$\sqrt[3]{1000x^3y^9} = \sqrt[3]{10^3x^3(y^3)^3}$$ Write each factor as a cube.

$$= \sqrt[3]{(10xy^3)^3}$$ Write as the cube of a product.

$$= 10xy^3$$ Simplify.

b. $\sqrt[4]{\dfrac{256g^8}{h^4k^{16}}}$

$$\sqrt[4]{\frac{256g^8}{h^4k^{16}}} = \sqrt[4]{\frac{4^4(g^2)^4}{h^4(k^4)^4}}$$ Write each factor as a power of 4.

$$= \sqrt[4]{\left(\frac{4g^2}{hk^4}\right)^4}$$ Write as the fourth power of a quotient.

$$= \frac{4g^2}{|h|k^4}$$ Simplify.

The absolute value symbols are needed to ensure the root is positive when h is negative. Note that $4g^2$ and k^4 are never negative.

Exercises

Simplify each radical expression. Use absolute value symbols when needed.

13. $\sqrt{36x^2}$ $\quad 6|x|$

14. $\sqrt[3]{216y^3}$ $\quad 6y$

15. $\sqrt{\dfrac{1}{100x^2}}$ $\quad \dfrac{1}{10|x|}$

16. $\dfrac{\sqrt{x^{20}}}{\sqrt{y^8}}$ $\quad \dfrac{x^{10}}{y^4}$

17. $\sqrt[3]{\dfrac{(x+3)^3}{(x-4)^6}}$ $\quad \dfrac{x+3}{(x-4)^2}$

18. $\sqrt[5]{x^{10}y^{15}z^5}$ $\quad x^2y^3z$

19. $\sqrt[3]{\dfrac{27z^3}{(z+12)^6}}$ $\quad \dfrac{3z}{(z+12)^2}$

20. $\sqrt[4]{2401x^{12}}$ $\quad 7|x^3|$

21. $\sqrt[3]{\dfrac{1331}{x^3}}$ $\quad \dfrac{11}{x}$

22. $\sqrt[4]{\dfrac{(y-4)^8}{(z+9)^4}}$ $\quad \dfrac{(y-4)^2}{|z+9|}$

23. $\sqrt[3]{\dfrac{a^6b^6}{c^3}}$ $\quad \dfrac{a^2b^2}{c}$

24. $\sqrt[3]{-x^3y^6}$ $\quad -xy^2$