6-7 Reteaching

Inverse Relations and Functions

- Inverse operations "undo" each other. Addition and subtraction are inverse operations. So are multiplication and division. The inverse of cubing a number is taking its cube root.
- If two functions are inverses, they consist of inverse operations performed in the opposite order.

Problem

What is the inverse of the relation described by $f(x) = x + 1$?

$f(x) = x + 1$	
$y = x + 1$	Rewrite the equation using *y*, if necessary.
$x = y + 1$	Interchange *x* and *y*.
$x - 1 = y$	Solve for *y*.
$y = x - 1$	The resulting function is the inverse of the original function.

So, $f^{-1}(x) = x - 1$.

Exercises

Find the inverse of each function.

1. $y = 4x - 5$
$f^{-1} = \frac{x+5}{4}$

2. $y = 3x^3 + 2$
$f^{-1} = \sqrt[3]{\frac{x-2}{3}}$

3. $y = (x + 1)^3$
$f^{-1} = \sqrt[3]{x} - 1$

4. $y = 0.5x + 2$
$f^{-1} = 2x - 4$

5. $f(x) = x + 3$
$f^{-1}(x) = x - 3$

6. $f(x) = 2(x - 2)$
$f^{-1}(x) = \frac{x+4}{2}$

7. $f(x) = \frac{x}{5}$
$f^{-1}(x) = 5x$

8. $f(x) = 4x + 2$
$f^{-1}(x) = \frac{x-2}{4}$

9. $y = x$
$f^{-1} = x$

10. $y = x - 3$
$f^{-1} = x + 3$

11. $y = \frac{x-1}{2}$
$f^{-1} = 2x + 1$

12. $y = x^3 - 8$
$f^{-1} = \sqrt[3]{x+8}$

13. $f(x) = \sqrt{x+2}$
$f^{-1}(x) = x^2 - 2$ for $x \geq -2$

14. $f(x) = \frac{2}{3}x - 1$
$f^{-1}(x) = \frac{3}{2}(x + 1)$

15. $f(x) = \frac{x+3}{5}$
$f^{-1}(x) = 5x - 3$

16. $f(x) = 2(x - 5)^2$
$f^{-1}(x) = 5 \pm \sqrt{\frac{x}{2}}$

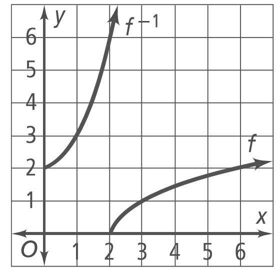
17. $y = \sqrt{x} + 4$
$f^{-1} = (x - 4)^2$ for $x \geq 0$

18. $y = 8x + 1$
$f^{-1} = \frac{x-1}{8}$

6-7 Reteaching (continued)

Inverse Relations and Functions

Examine the graphs of $f(x) = \sqrt{x - 2}$ and its inverse, $f^{-1}(x) = x^2 + 2$, at the right.



Notice that the range of f and the domain of f^{-1} are the same: the set of all real numbers $x \geq 0$.

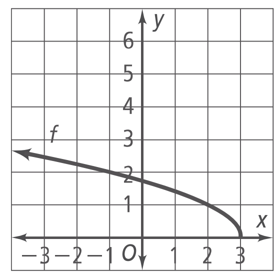
Similarly, the domain of f and the range of f^{-1} are the same: the set of all real numbers $x \geq 2$.

This inverse relationship is true for all relations whenever both f and f^{-1} are defined.

Problem

What are the domain and range of the inverse of the function $f(x) = \sqrt{3 - x}$?

f is defined for $3 - x \geq 0$ or $x \leq 3$.



Therefore, the domain of f and the range of f^{-1} is the set of all $x \leq 3$.

The range of f is the set of all $x \geq 0$. So, the domain of f^{-1} is the set of all $x \geq 0$.

Exercises

Name the domain and range of the inverse of the function.

19. $y = 2x - 1$
The domain and the range is the set of all real numbers.

20. $y = 2 - \frac{1}{x}$
domain: $x \neq 2$; range: $y \neq 0$

21. $y = \sqrt{x + 5}$
domain: $x \geq 0$; range: $y \geq -5$

22. $y = \sqrt{-x} + 8$
domain: $x \geq 8$; range: $y \leq 0$

23. $y = 3\sqrt{x} + 2$
domain: $x \geq 2$; range: $y \geq 0$

24. $y = (x - 6)^2$
domain: $x \geq 0$; range: all real numbers

25. $y = x^2 - 6$
domain: $x \geq -6$; range: all real numbers

26. $y = \frac{1}{x + 4}$
domain: $x \neq 0$; range: $y \neq -4$

27. $y = \frac{1}{(x + 4)^2}$
domain: $x > 0$; range: $y \neq -4$