UNIT 4 LESSON 1

PRECALCULUS B

LESSON:

POLAR COORDINATES

RECTANGULAR COORDINATES vs POLAR COORDINATES

(x, y) (r, θ)

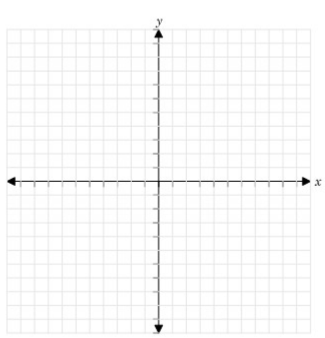
Position from center:

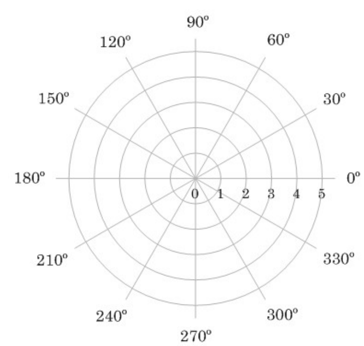
(horizontal, vertical) (radial, angle)

Like: latitude & longitude **Like: radar**

RECTANGULAR COORDINATES vs POLAR COORDINATES

(x, y) (r, θ)

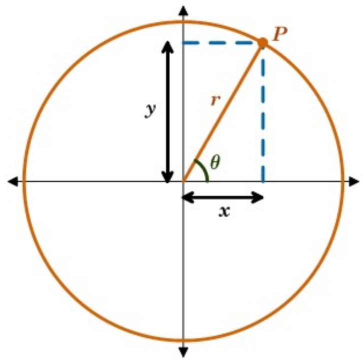




… like the Unit Circle, but bigger!!

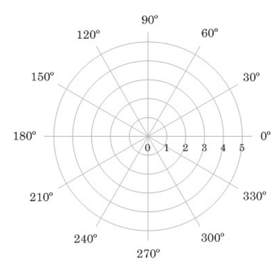
RECTANGULAR COORDINATES vs POLAR COORDINATES

(x, y) (r, θ)



Each system is useful ☺
And both ways can get you to the same point!

POLAR COORDINATES

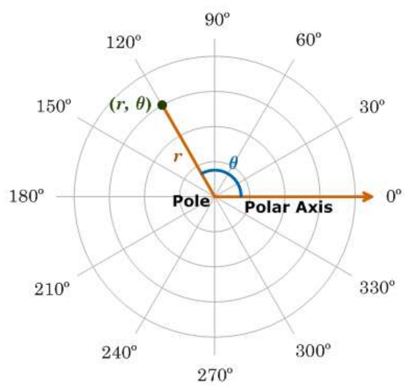


Pole – center point
Polar Axis – the base line for measuring the angle from
Radial Lines – the lines to measure the distance from the center
Polar Coordinates – the ordered pair (r, θ)
r – the distance from the pole
Θ – the angle from the polar axis – either in degrees or radians

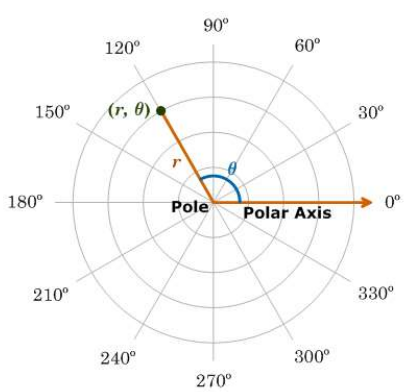
POLAR COORDINATES

Polar Coordinates

Point $P\ (r,\theta)$ is located a directed distance, r, from the pole at an angle of rotation, θ, from the polar axis.



POLAR COORDINATES

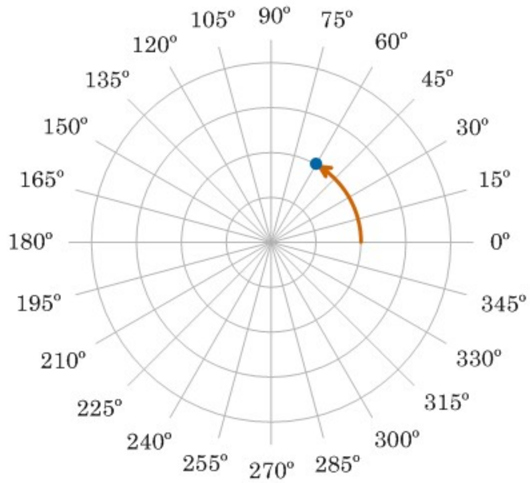


So this point is at

(3, 120°)

POLAR COORDINATES

What are the coordinates of this point?

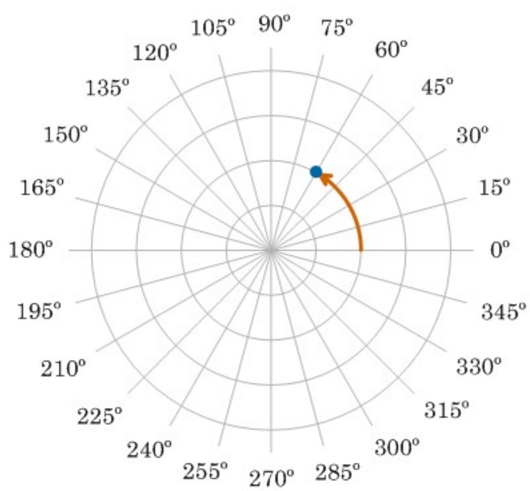


POLAR COORDINATES

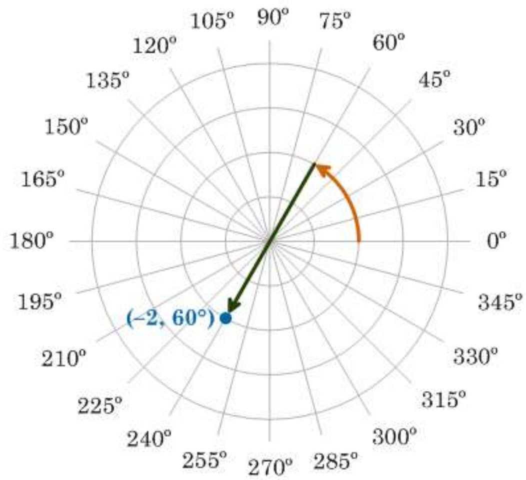
What are the coordinates of this point?

(2, 60°)

Can the first number be negative?



POLAR COORDINATES



Can the first number be negative?

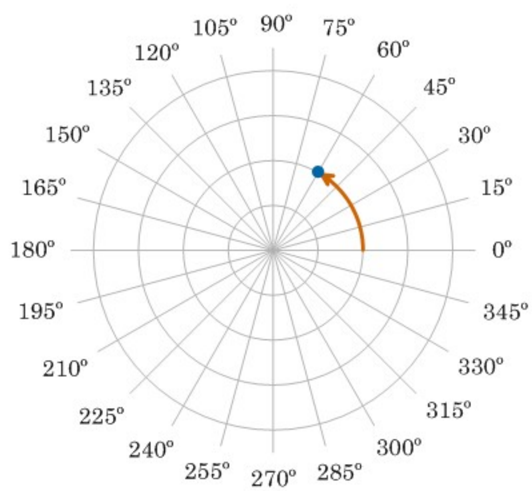
Yes!

It rotates the point 180°

POLAR COORDINATES

Since this is based on a circle, there are actually infinite ways to write the coordinates of any point!

Just rotate around back to the point ☺

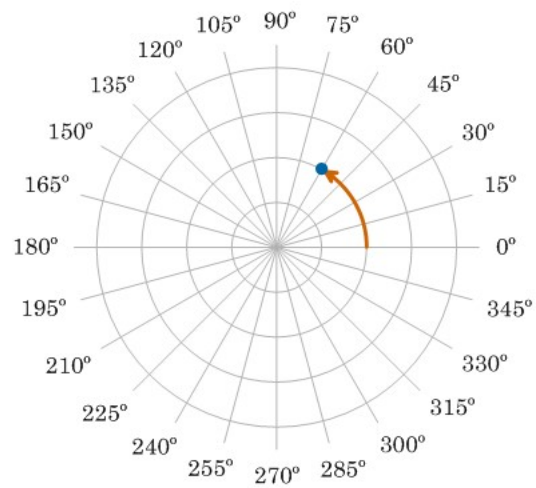


POLAR COORDINATES

So . . .

(2, 60)
Add 360°
(2, 420)
Another 360°
(2, 780)
Or subtract 360°
(2, -300)

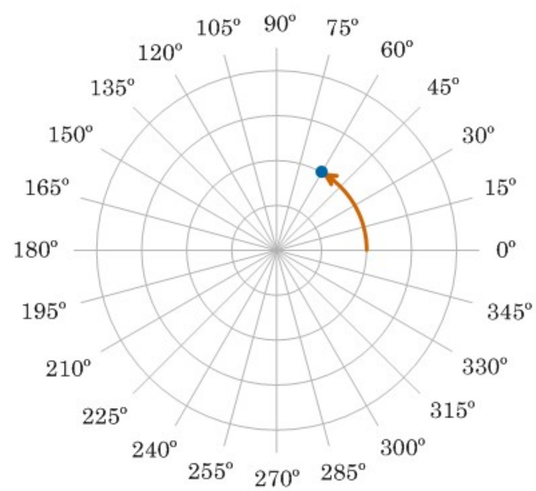
Etc.



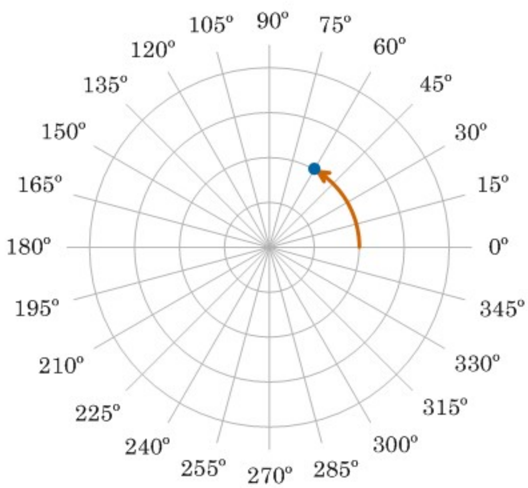
POLAR COORDINATES

Or change the sign on the radial point and add 180° . . .

Starting with
(2, 60)
Change r sign
& add 180°
(-2, 240)
Another 360°
(-2, 600)



POLAR COORDINATES



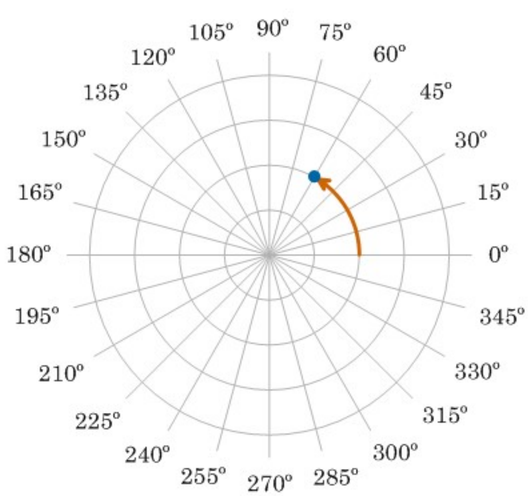
Now let's change it to radians . . .

Check the Unit Circle:
$60° = \pi/3$

So, (2, 60°)

Is also (2, $\pi/3$)

POLAR COORDINATES



Variations work the same, except add 2π instead of 360°

(2, $\pi/3$)

(2, $7\pi/3$)

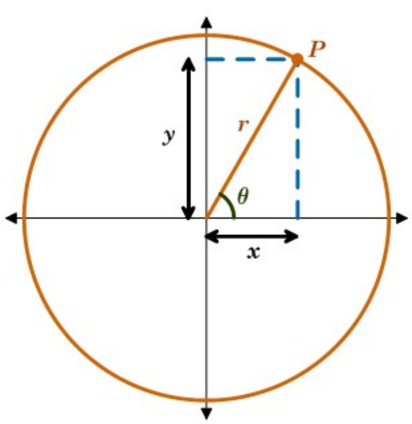
(2, $13\pi/3$)

Etc.

POLAR COORDINATES

	Degrees	Radians
Standard	(2, 60)	(2, π/3)
Add 1 rotation	(2, 420)	(2, 7π/3)
Add 2 rotations	(2, 780)	(2, 13π/3)
Subtract 1 rotation	(2, -300)	(2, -5π/3)
Change the radial sign & add a half rotation, that is either 180° or 1π	(-2, 240)	(-2, 4π/3)

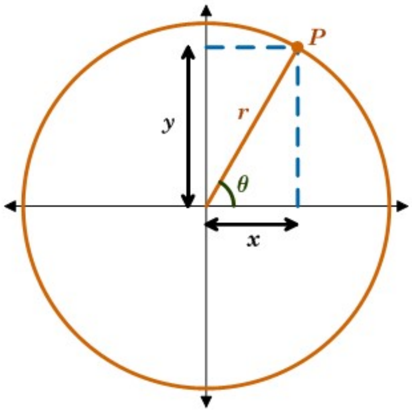
POLAR COORDINATES – Converting to Rectangular (x, y)



Remember, on the Unit Circle, that the x-coordinate is cos θ, and the y-coordinate is sin θ.

But now the distance from the center is not always 1 . . .

POLAR COORDINATES – Converting to Rectangular

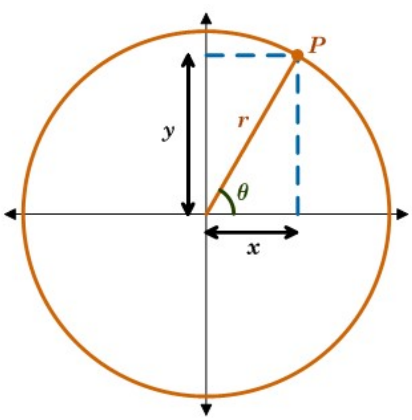


So, multiply by
the distance from
the center . . .

x = r cosθ

y = r sinθ

POLAR COORDINATES – Converting to Rectangular



TRY IT:

Convert
(3, π/6)

to rectangular
(x, y)

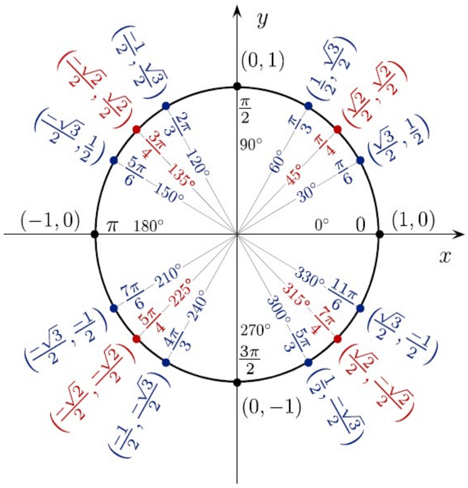
$$x = r\cos\theta$$

$$y = r\sin\theta$$

POLAR COORDINATES – Converting to Rectangular

$x = r\cos\theta$

$y = r\sin\theta$



Remember, use the Unit Circle for sine & cosine!

Polar $\left(3, \frac{\pi}{6}\right)$

$x = 3\cos\frac{\pi}{6}$

$x = 3\left(\frac{\sqrt{3}}{2}\right)$

$x = \frac{3\sqrt{3}}{2}$

$y = 3\sin\frac{\pi}{6}$

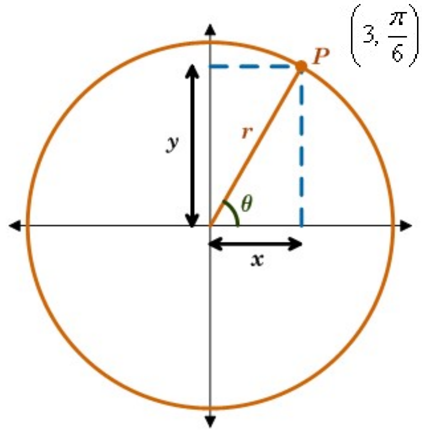
$y = 3\left(\frac{1}{2}\right)$

$y = \frac{3}{2}$

Rectangular $\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$

POLAR COORDINATES – Converting to Rectangular

$\left(3, \frac{\pi}{6}\right)$ or $\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$

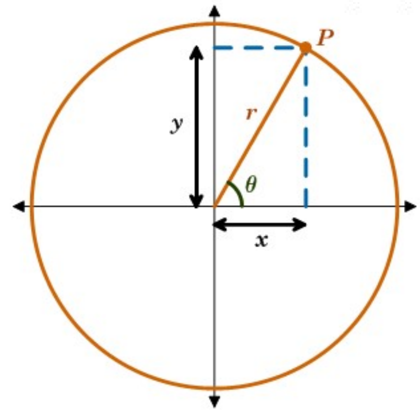


AND . . . Converting Rectangular to Polar

$r = \sqrt{x^2 + y^2}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$, where $x \neq 0$

(1, 3)



How To Get the Conversion Rule for the Radius:

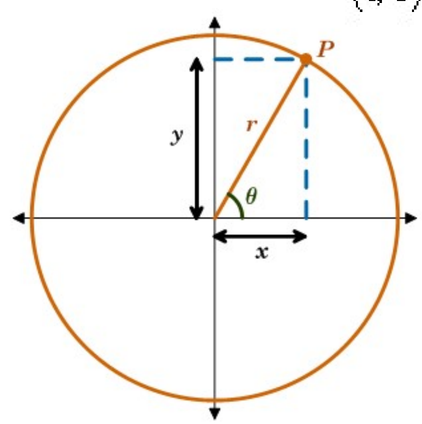
Remember the Pythagorean Theorem: $a^2 + b^2 = c^2$
Use x and y for a and b, then use r for c (the hypotenuse).
$x^2 + y^2 = r^2$
Then square root both sides to solve for r!

AND . . . Converting Rectangular to Polar

$r = \sqrt{x^2 + y^2}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$, where $x \neq 0$

(1, 3)



How To Get the Conversion Rule for the Angle:

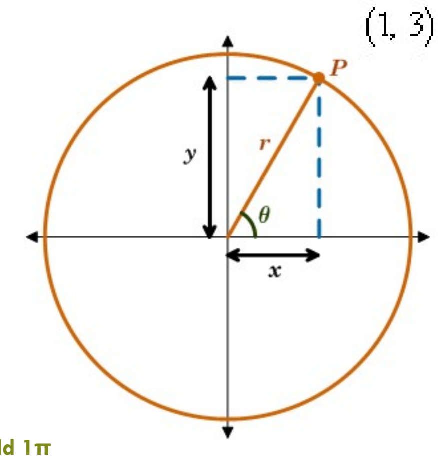
See that x is the adjacent to Θ and y is the opposite to Θ.
So we can set it up as tan Θ = y/x.
Then solve for Θ by using inverse tangent!

AND . . . Converting Rectangular to Polar

TRY it for (1, 3) … check if your calculator is set to radians!

$$r=\sqrt{x^2+y^2}$$
$$r=\sqrt{1^2+3^2}$$
$$r=\sqrt{10}\approx 3.162$$

$$\theta=\tan^{-1}\left(\frac{y}{x}\right)$$
$$\theta=\tan^{-1}\left(\frac{3}{1}\right)$$
$$\theta\approx 1.249 \text{ radians}$$



The exact polar coordinates are:
(√10, arctan3)
The approximate coordinates are:
(3.162, 1.249)
Or in degrees:
(3.162, 71.565)

Or add another 2π, about 6.28
(3.162, 7.529)
Or start with - r, which means add 1π
(-3.162, 4.391)
Etc.

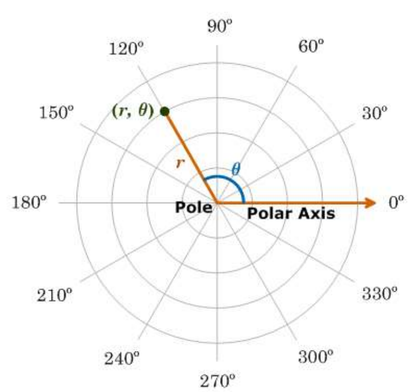
POLAR COORDINATES Summary:

Polar Coordinates

Point $P\ (r,\theta)$ is located a directed distance, r, from the pole at an angle of rotation, θ, from the polar axis.

$$r=\sqrt{x^2+y^2}$$

$$\theta=\tan^{-1}\left(\frac{y}{x}\right), \text{ where } x\neq 0$$



$$x=r\cos\theta$$

$$y=r\sin\theta$$

Questions??

Review the Key Terms and Key Concepts documents for this unit.

Look up the topic at khanacademy.org

Come to Open Office time to ask me.
Check your Planner for the day & time.

Reserve a time for a call with me at jpattersonmath.youcanbook.me

We can use the LiveLesson whiteboard to go over problems together!