UNIT 3 Lessons 1-4

PRECALCULUS A

LESSONS:

- Analyzing Functions
- Even & Odd Functions
- Asymptotes and End Behavior
- Continuous & Discontinuous Functions

our class website: nca-patterson.weebly.com

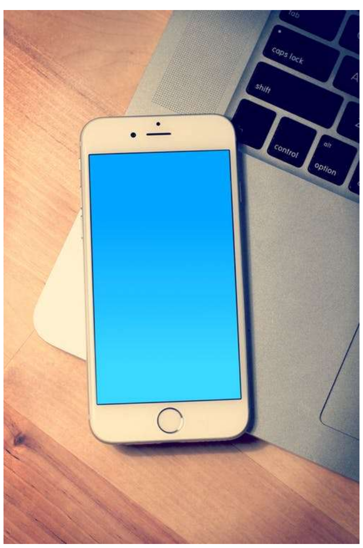
book a call time: jpattersonmath.youcanbook.me

WHY?

Why do we analyze so many features of functions??

Well, the more we understand the specifications of something, the more informed we are to make good decisions.

. . . financial investments, medical research, recipes, etc.



Like with buying a new phone:

- Storage
- Memory
- Speed
- Battery life
- Ports
- Screen size

… it helps to understand the features.

So, here are some “specs” for functions . . .

Key Concept

Increasing, Decreasing, and Constant Intervals

- A function is increasing on an open interval in which $f(x_1) < f(x_2)$ when $x_1 < x_2$ for all x_1 and x_2 in the interval.
- A function is decreasing on an open interval in which $f(x_1) > f(x_2)$ when $x_1 < x_2$ for all x_1 and x_2 in the interval
- A function is constant on an open interval in which $f(x_1) = f(x_2)$ for all x_1 and x_2 in the interval.

In other words:

Key Concept

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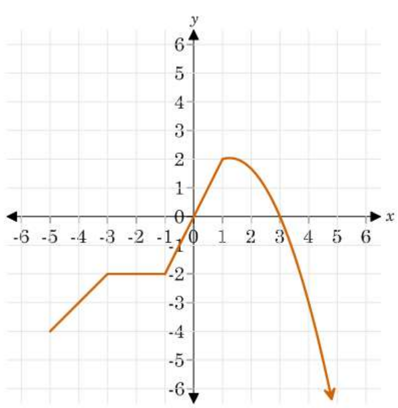
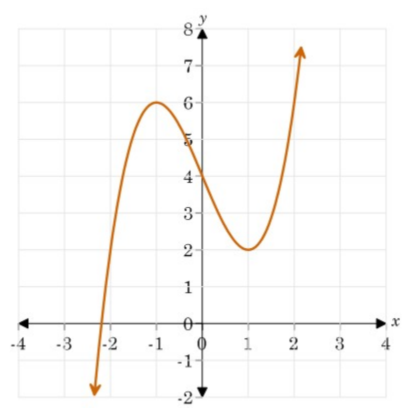
- A function is constant on an open interval in which $f(x_1) = f(x_2)$ for all x_1 and x_2 in the interval.

In other words:

As you go from left to right, does the graph go up, down, or stay level.

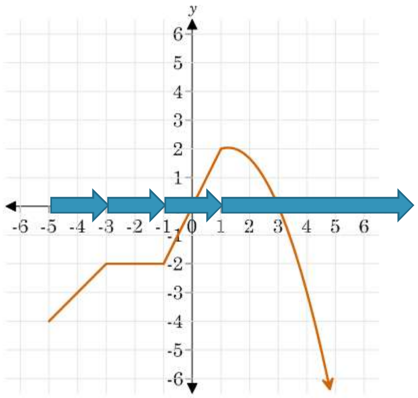
Are these increasing, decreasing, and/or constant?

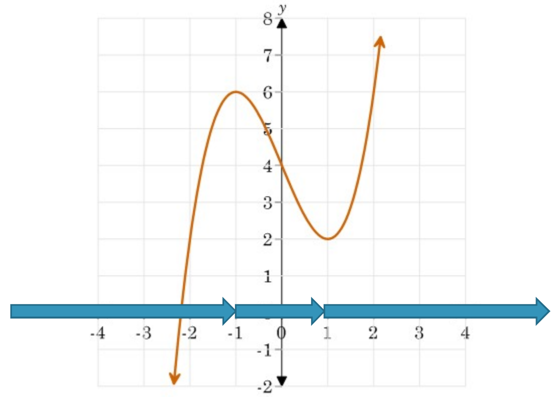




Identify the specific domain intervals (look at the end points) for each increasing, decreasing, or constant section.

Intervals

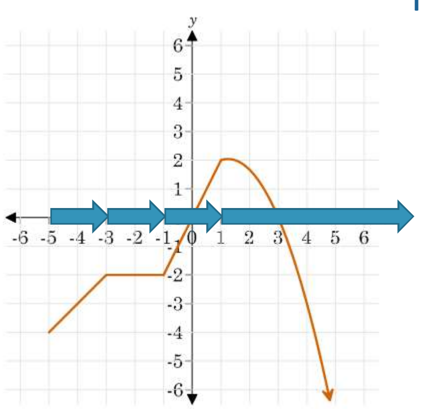


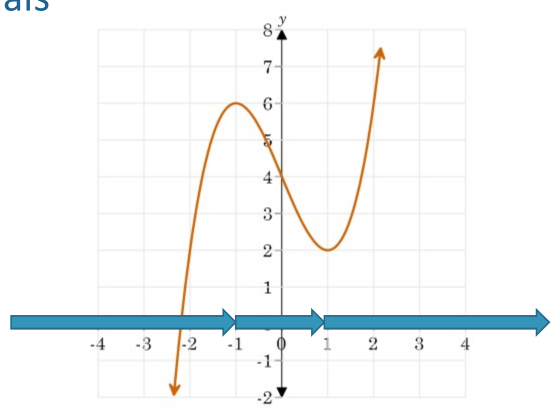


Increasing __________
Decreasing _________
Constant ___________

Increasing ___________
Decreasing __________
Constant ____________

Intervals

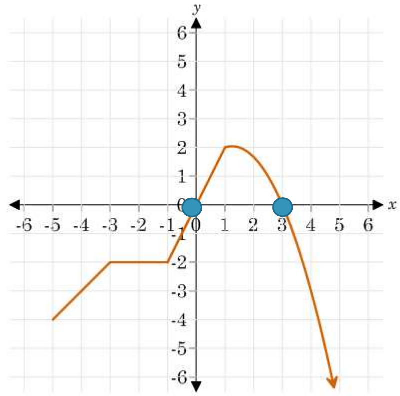


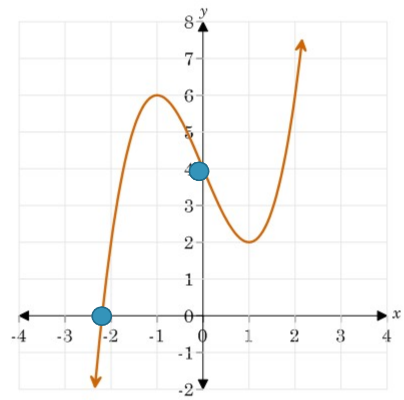


Increasing (-5,-3)U(-1,1)
Decreasing (1,∞)
Constant (-3,-1)

Increasing (-∞,-1)U(1, ∞)
Decreasing (-1,1)
Constant never

We can also identify the x and y intercepts, or look at other requested points or regions.





Remember, the x-intercept is where y=0, and the y-intercept is where x=0.

Key Concept

Even and Odd Functions

- A function f is even if for each value of x in the domain, $f(-x) = f(x)$. The graph of an even function displays symmetry with respect to the y-axis; if the point (x, y) lies on the graph of f, then the point $(-x, y)$ also lies on the graph of f.
- A function f is odd if for each value of x in the domain, $f(-x) = -f(x)$. The graph of an odd function displays symmetry with respect to the origin; if the point (x, y) lies on the graph of f, then the point $(-x, -y)$ also lies on the graph of f.

In other words:

Key Concept

Even and Odd Functions

- A function f is even if for each value of x in the domain, $f(-x) = f(x)$. The graph of an even function displays symmetry with respect to the y-axis; if the point (x, y) lies on the graph of f, then the point $(-x, y)$ also lies on the graph of f.
- A function f is odd if for each value of x in the domain, $f(-x) = -f(x)$. The graph of an odd function displays symmetry with respect to the origin; if the point (x, y) lies on the graph of f, then the point $(-x, -y)$ also lies on the graph of f.

In other words:

Even or odd functions are just looking for specific kinds of symmetry.
If a function doesn't have one of these kinds of symmetry, then it is neither even nor odd.

Key Concept

Test for Even or Odd Functions

To test if a function $f(x)$ is even, odd, or neither, substitute $-x$ for x and simplify.

- If $f(-x) = f(x)$, then the function is even.
- If $f(-x) = -f(x)$, then the function is odd.
- Otherwise, the function is neither even nor odd.

EVEN:

If it is symmetrical across the y-axis, plugging in x or its opposite will result in the same value for y.

An even function with the point (2,-3) would also have the point (-2, -3).

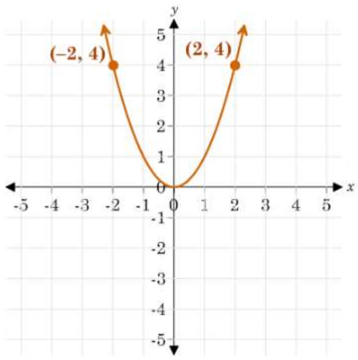
ODD:

If it is symmetrical with respect to the origin (0,0) plugging in the opposite of x will result in the opposite value for y.

An odd function with the point (2,-3) would also have the point (-2,3).

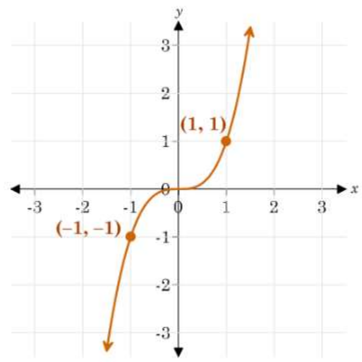
Classic Examples of Even or Odd Functions

Even: y=x^2



"fold symmetry"
… but only folding at the y-axis

Odd: y=x^3



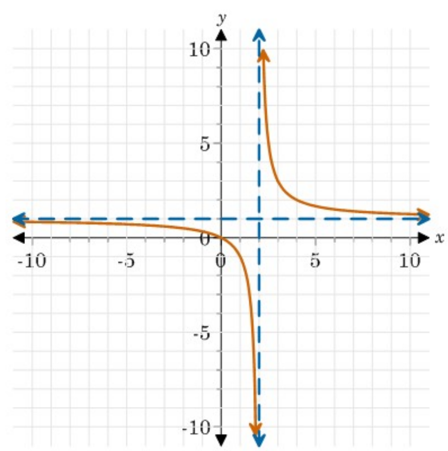
"spin 180 symmetry"
… but only around the origin

Key Concept

Vertical and Horizontal Asymptotes

The line $x = a$ is a vertical asymptote of the graph of $f(x)$ if $f(x) \to \infty$ or $f(x) \to -\infty$ as $x \to a$.

The line $y = b$ is a horizontal asymptote of the graph $f(x)$ if $f(x) \to b$ as $x \to \infty$ or $x \to -\infty$.



In other words:

Key Concept

Vertical and Horizontal Asymptotes

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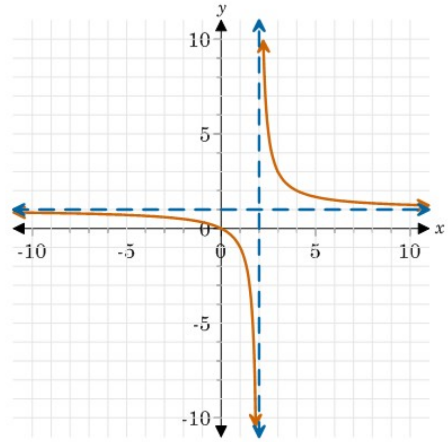


In other words:

An asymptote line is like a wall that when you approach it, you turn quick to miss it.

End Behavior Notation

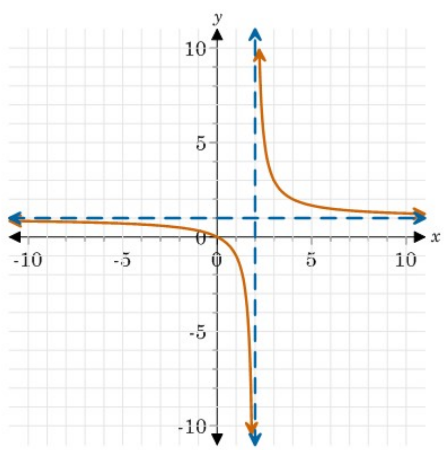
Vertical Asymptote x=2

As x approaches the asymptote from the negative side, y turns down.

Notation??

As x approaches the asymptote from the positive side, y turns up.

Notation??



Horizontal Asymptote y=1

End Behavior Notation

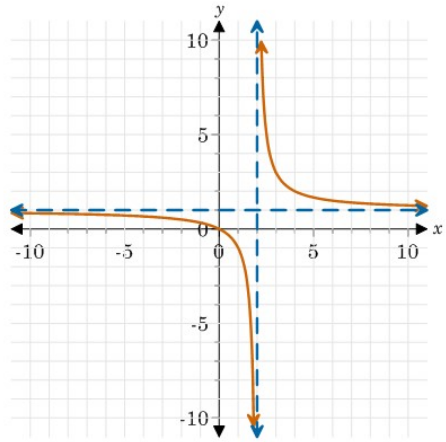
Vertical Asymptote x=2

As x approaches the asymptote from the negative side, y turns down.

f(x) → -∞ as x→ 2-

As x approaches the asymptote from the positive side, y turns up.

f(x) → +∞ as x→ 2+



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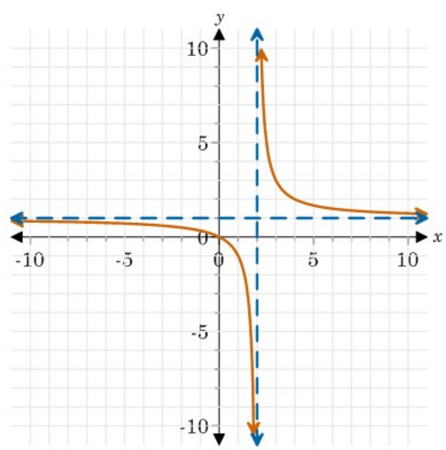

Horizontal Asymptote y=1

As x turns around and approaches positive infinity, y approaches +1.

Notation??

As x turns around and approaches negative infinity, y approaches -1.

Notation??

End Behavior Notation

Vertical Asymptote x=2

As x approaches the asymptote from the negative side, y turns down.

f(x) → -∞ as x→ 2-

As x approaches the asymptote from the positive side, y turns up.

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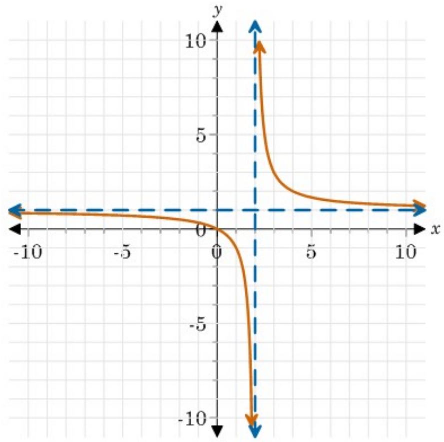


Horizontal Asymptote y=1

As x turns around and approaches positive infinity, y approaches +1.

f(x) → 1 as x→ +∞

As x turns around and approaches negative infinity, y approaches -1.

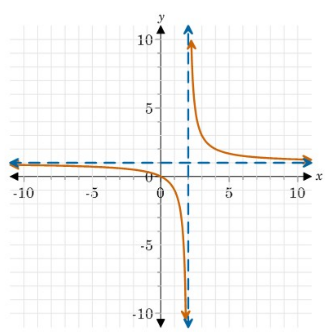
f(x) → 1 as x→ -∞

End Behavior Notation

Vertical Asymptote x=2

f(x) → -∞ as x→ 2-

f(x) → +∞ as x→ 2+



Horizontal Asymptote y=1

f(x) → 1 as x→ +∞

f(x) → -1 as x→ -∞

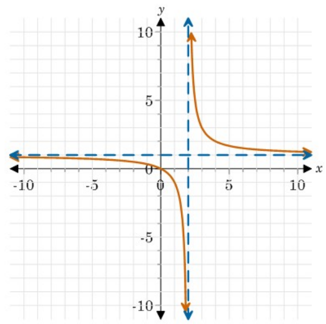
In other words:

End Behavior Notation

Vertical Asymptote x=2

f(x) → -∞ as x→ 2-

f(x) → +∞ as x→ 2+



Horizontal Asymptote y=1

f(x) → 1 as x→ +∞

f(x) → -1 as x→ -∞

In other words:

As the function heads out in all directions, does it have to watch out for any walls?

Key Concept

Continuous and Discontinuous Functions

A function is continuous if its graph is a single, unbroken curve.

A function is discontinuous if its graph has a hole, jump, or vertical asymptote.

In other words:

Key Concept

Continuous and Discontinuous Functions

A function is continuous if its graph is a single, unbroken curve.

A function is discontinuous if its graph has a hole, jump, or vertical asymptote.

In other words:

If your pencil would continuously stay on the paper to draw the function … it is continuous.

If you have to lift your pencil to draw another part … it is discontinuous.

Pretty simple . . .

But there are different types of breaks you can get with a discontinuous function.

Discontinuity Types
1. Removable Discontinuity
2. Nonremovable Discontinuity
    a. Infinite Discontinuity
    b. Jump Discontinuity

***And you can have more than one type of discontinuity in a discontinuous function.**

Continuous

DRAW

Nonremovable Infinite Discontinuity

… in other words, a vertical asymptote

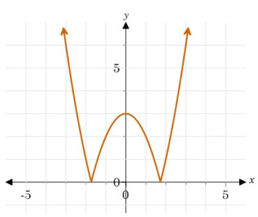
Removable Discontinuity

… in other words a removed point

Nonremovable Jump Discontinuity

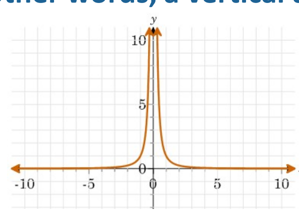
… in other words, it breaks and jumps

Continuous



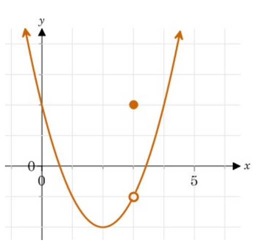
Nonremovable Infinite Discontinuity

… in other words, a vertical asymptote



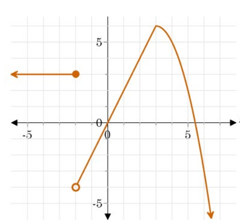
Removable Discontinuity

… in other words a removed point



Nonremovable Jump Discontinuity

… in other words, it breaks and jumps



Function Specs so far …

- Intervals that are increasing, decreasing, or constant
- Intercepts at the x-axis and/or y-axis
- Even or Odd type symmetry, or neither
- Horizontal and/or Vertical Asymptotes, or none
- End Behavior
- Continuous or Discontinuous
- Types of Discontinuity – removable (point), non-removable (infinite at a vertical asymptote, or jump)
- And, we may be asked to look at other regions or points of interest.

Questions??

Review the Key Terms and Key Concepts documents for this unit.

Look up the topic at khanacademy.org and virtualnerd.com

Check our class website at nca-patterson.weebly.com

*Reserve a time for a call with me at
jpattersonmath.youcanbook.me
We can use the LiveLesson whiteboard
to go over problems together.