UNIT 3 Lessons 7-8

PRECALCULUS A

LESSONS:

- Transformations of Functions
- Multiple Transformations of Functions

our class website: nca-patterson.weebly.com

book a call time: jpattersonmath.youcanbook.me

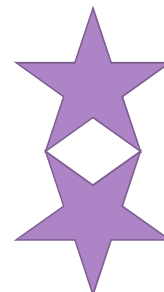**Remember these from Algebra 2 . . .**

REFLECT

SHIFT →

STRETCH

COMPRESS

Vertical Shift:
Add or Subtract
To the Function

$$y = f(x) + a$$

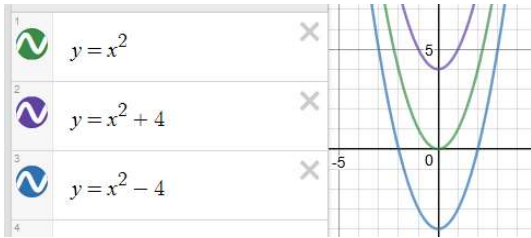**If +a, then shifts up**
If −a, then shifts down

Horizontal Shift:
Add or Subtract
To the Variable

$$y = f(x + a)$$

If +a, then shifts left
If −a, then shifts right
. . . I know, it seems
backwards

Vertical Shift:
Add or Subtract
To the Function
$y = f(x) + a$

If +a, then shifts up
If –a, then shifts down

$y = x^2$

$y = x^2 + 4$

$y = x^2 - 4$

Horizontal Shift:
Add or Subtract
To the Variable
$y = f(x + a)$

If +a, then shifts left
If –a, then shifts right

$y = x^2$

$y = (x + 4)^2$

$y = (x - 4)^2$
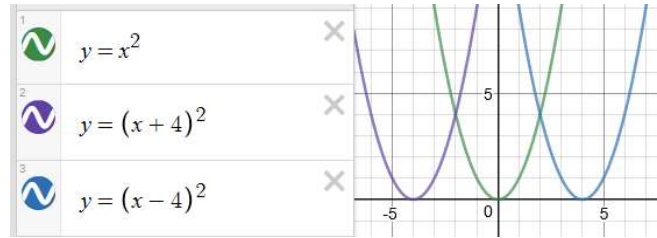
Practice:

y = x^3 + 5

y = (x - 2)^4

y = (x + 4)^3

y = x^2 - 6

Practice:

y = x^3 + 5   vertical shift 5 up

y = (x-2)^4   horizontal shift 2 right

y = (x+4)^3   horizontal shift 4 left

y = x^2 − 6   vertical shift 5 down

Vertical Stretch/Compress: Multiply the Function

$$y = af(x)$$

If |a|>1, then it stretches toward the y-axis & away from the x-axis.

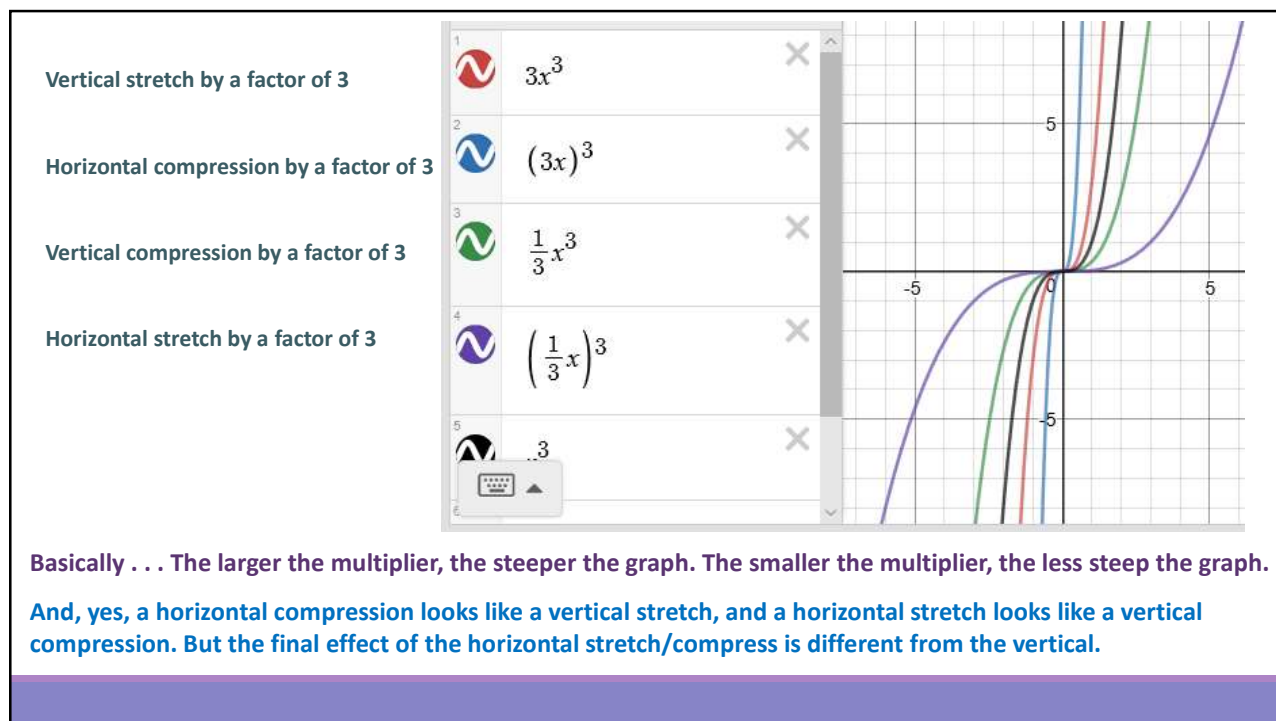**If 0<|a|<1, then it compresses away from the y-axis & toward the x-axis.**

Horizontal Stretch/Compress: Multiply the Variable

$$y = f(ax)$$

If |a|>1, then it compresses away from the x-axis & toward the y-axis.

If 0<|a|<1, then it stretches toward the x-axis & away from the y-axis.

. . . I know, it seems backwards

Vertical stretch by a factor of 3

$3x^3$

Horizontal compression by a factor of 3

$(3x)^3$

Vertical compression by a factor of 3

$\frac{1}{3}x^3$

Horizontal stretch by a factor of 3

$\left(\frac{1}{3}x\right)^3$

$.3$

Basically . . . The larger the multiplier, the steeper the graph. The smaller the multiplier, the less steep the graph.

And, yes, a horizontal compression looks like a vertical stretch, and a horizontal stretch looks like a vertical compression. But the final effect of the horizontal stretch/compress is different from the vertical.

Vertical Reflection
Across the x-axis
Negative on the Function

$$y = -f(x)$$

What was above, goes below.
What was below, goes above.
But all at the same distance it was from the x-axis.

Horizontal Reflection
Across the y-axis
Negative on the Variable

$$y = f(-x)$$

What was to the right, goes left.
What was to the left, goes right
But all at the same distance it was from the y-axis.

Vertical Reflection
Across the x-axis
Negative on the Function
   y = -f(x)

What was above, goes below.
What was below, goes above.
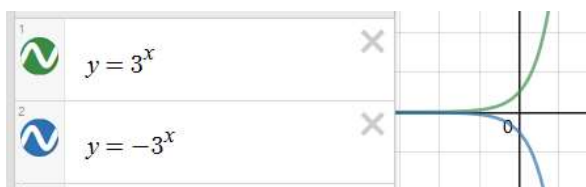But all at the same distance it was
from the x-axis.

$y = 3^x$

$y = -3^x$

Horizontal Reflection
Across the y-axis
Negative on the Variable
   y = f(-x)

What was to the right, goes left.
What was to the left, goes right
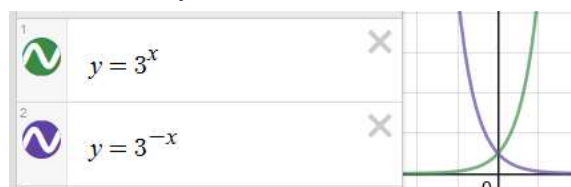But all at the same distance it was
from the y-axis.

$y = 3^x$

$y = 3^{-x}$

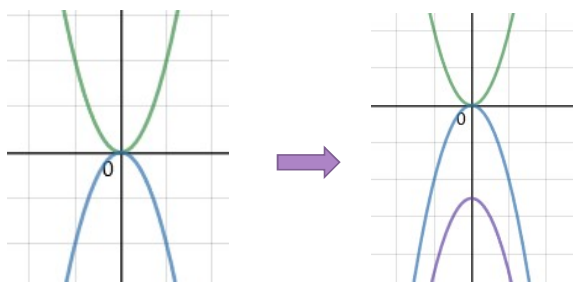**More than one transformation in the same equation? Yes!**

But order matters!!

Key Concept

Order of Transformations of Functions

When carrying out multiple transformations of a function, perform them
in the following order:

1. horizontal shift
2. stretch or compression
3. reflection
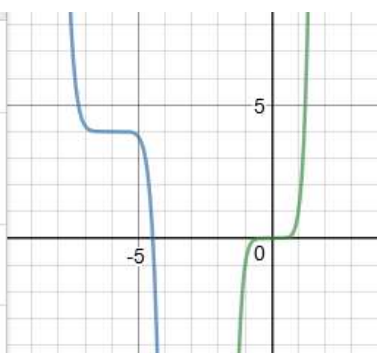4. vertical shift

Correct:
reflect, then
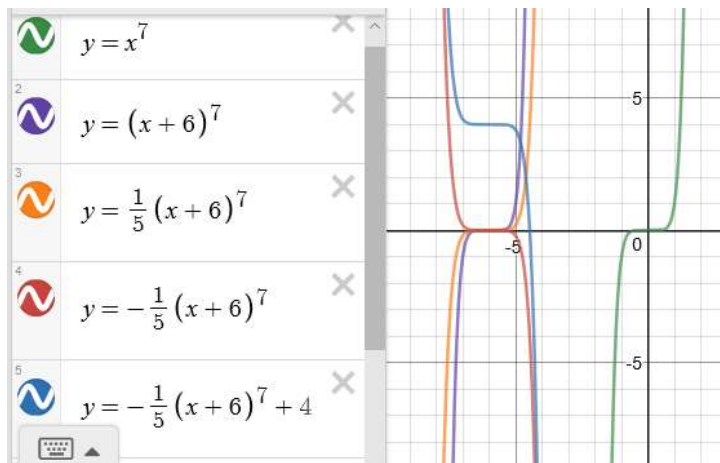vertical shift

Incorrect:
vertical shift,
then reflect

Name all of the transformations, in proper order:

$$y = x^7$$

$$y = -\frac{1}{5}(x+6)^7 + 4$$

Name all of the transformations, in proper order:

$y = x^7$

$y = (x+6)^7$

$y = \frac{1}{5}(x+6)^7$

$y = -\frac{1}{5}(x+6)^7$

$y = -\frac{1}{5}(x+6)^7 + 4$

Parent Function

Horizontal shift
6 to the left

Vertical compression
By a factor of 5

Reflection over the
x-axis

Vertical shift
4 up

Questions??

Review the **Key Terms** and **Key Concepts** documents for this unit.

Look up the topic at **khanacademy.org** and **virtualnerd.com**

Check our class website at **nca-patterson.weebly.com**

*Reserve a time for a call with me at
jpattersonmath.youcanbook.me
We can use the LiveLesson whiteboard
to go over problems together.