UNIT 4 LESSONS 1-4

PRECALCULUS A

LESSONS:

- Polynomial Functions
- Real Zeros of a Polynomial Function
- Dividing Polynomials
- Complex Zeros of a Polynomial Function

… lots of vocabulary today ☺

What is a polynomial function??

What is a polynomial function??

Basically,
it is a set of terms with the same variable, but each term can have a different power on the variable.

For example: $f(x) = 8x^3 - 3x^2 + 4x + 3$

**Remember, terms are separated by plus and minus signs.
So, how many terms are in this example?

Polynomial functions:

So where is the variable on the last term?

For example: $f(x) = 8x^3 - 3x^2 + 4x + 3$

Polynomial functions:

So where is the variable on the last term?

For example: $f(x) = 8x^3 - 3x^2 + 4x + 3$

Yes, that last term has a variable in it, too!
It has x^0, which equals 1, so we don't have
to write it. ☺

BTW, the powers are all positive whole numbers.
This makes sure there are no variables in a
denominator, and no roots of the variable.

The Degree of a Polynomial Function:

This is the largest power on the variable.

For example, this has a degree of 3:

$$f(x) = 8x^3 - 3x^2 + 4x + 3$$

… and it will be very important to know what the degree is

… this should be ringing a bell from Algebra 2

Standard Form of a Polynomial Function:

Write the terms starting with the one with the largest power down to the one with the smallest power. This makes identifying the degree easier!

For example, this is in standard form.

$$f(x) = 8x^3 - 3x^2 + 4x + 3$$

And this one is not in standard form.

$$f(x) = 5 - 4x^2 - 3x^3$$

More about Polynomial Functions:

The "leading coefficient" is the number in front of the variable in the "leading term" when the polynomial is written in "standard form"

For example, this has a leading coefficient of 8:

$$f(x) = 8x^3 - 3x^2 + 4x + 3$$

More about Polynomial Functions:

The domain of a polynomial is the set of all real numbers.

Think about it, domain restrictions come from denominators and roots, but we aren't allowing those.

See, no domain restrictions!

$$f(x) = 8x^3 - 3x^2 + 4x + 3$$

Time to practice . . .

1. Identify the leading term, the leading coefficient, and degree of each polynomial function.

a. $f(x) = 16 - x^2$
b. $f(x) = 6x^3 - 3x + 9$
c. $f(x) = 7x^6 - 3x^4 + x^2 - 11$
d. $f(x) = 4x^4 - 2x^7 - 5x^3 + x$

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a. **Leading term is –x^2, Leading coefficient it -1, Degree is 2**
b. **Leading term is 6x^3, Leading coefficient is 6, Degree is 3**
c. **Leading term is 7x^6, Leading coefficient is 7, Degree is 6**
d. **Leading term is -2x^7, Leading coefficient is -2, Degree is 7**

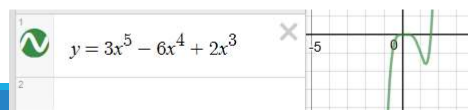
Back to End Behaviors:

Even Degree Functions go the same direction at both ends.
Odd Degree Functions go opposite directions at each end.

**NOTE: This is different from even or odd symmetry!

Even Degree … like y=x^2

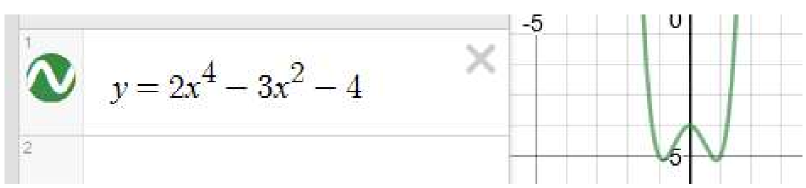
Odd Degree … like y=x^3



Even Degree End Behaviors:

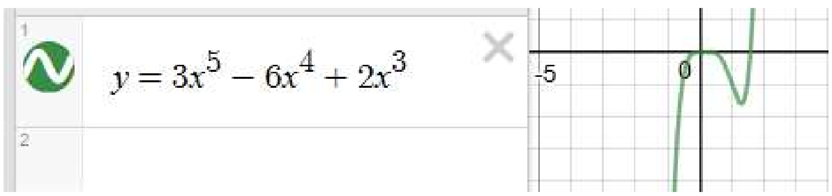
Even Degree Functions go the same direction at both ends.
If the leading coefficient is positive, both ends go up.
If the leading coefficient is negative, both ends do down.

… because the negative makes it a reflection!



Odd Degree End Behaviors:

Odd Degree Functions go opposite directions at each end.
If the leading coefficient is positive, it will go down on the left, and go up on the right.
If the leading coefficient is negative, it will go up on the left, and go down on the right.



Time to practice:

2. Describe the end behavior of each polynomial function.

a. $f(x) = 16 - x^2$
b. $f(x) = 6x^3 - 3x + 9$
c. $f(x) = 7x^6 - 3x^4 + x^2 - 11$
d. $f(x) = 4x^4 - 2x^7 - 5x^3 + x$

a. **Leading term is –x^2, Leading coefficient it -1, Degree is 2 … so**
b. **Leading term is 6x^3, Leading coefficient is 6, Degree is 3 … so**
c. **Leading term is 7x^6, Leading coefficient is 7, Degree is 6 … so**
d. **Leading term is -2x^7, Leading coefficient is -2, Degree is 7 … so**

Time to practice:

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d. $f(x) = 4x^4 - 2x^7 - 5x^3 + x$

a. … so even degree & negative coefficient means down and down
b. … so odd degree & positive coefficient means down on the left and up on the right
c. … so even degree & positive coefficient means up and up
d. … so odd degree & negative coefficient means up on the left and down on the right

Turning Points of a Polynomial Function:

The place, or places, where the graph switches from increasing to decreasing, or from decreasing to increasing, is called a "turning point".

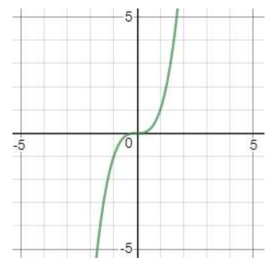
The maximum possible number of turning points is one less than the degree of the polynomial.

… aren't you glad someone checked this pattern for us!

Turning Points of a Polynomial Function:

NOTE: Sometimes there are zero turning points!!

For example, $y = x^3$
The bend in the middle is less increasing, but does continue to increase!



Time to practice:

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a. **Degree is 2 …** **so the maximum number of turning points is ????**
b. **Degree is 3 …** **so the maximum number of turning points is ????**
c. **Degree is 6 …** **so the maximum number of turning points is ????**
d. **Degree is 7 …** **so the maximum number of turning points is ????**

Time to practice:

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a. **Degree is 2 … so the maximum number of turning points is 1**
b. **Degree is 3 … so the maximum number of turning points is 2**
c. **Degree is 6 … so the maximum number of turning points is 5**
d. **Degree is 7 … so the maximum number of turning points is 6**

Speaking of maximums … and minimums

The place where a graph has a highest point is called a maximum.
The place where a graph has a lowest point is called a minimum.

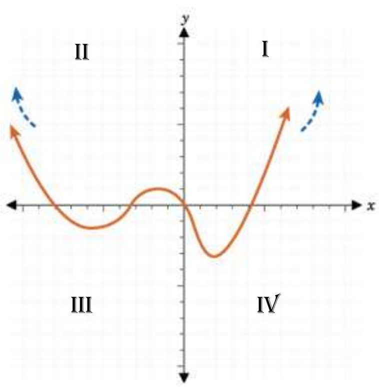
These are called Extreme Values, or, Extrema.

Global Extrema vs Local Extrema

Global means the entire graph.

Local means one section, or interval, of the graph.

Global Extrema vs Local Extrema



Global Maximum?

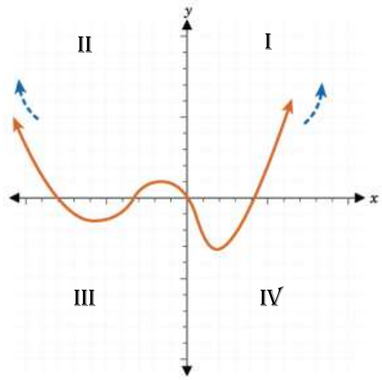
Global Minimum?

Local Maximums?
In which quadrants?

Local Minimums?
In which quadrants?

Roman numerals translation: I is 1, II is 2, III is 3, IV is 4.

Global Extrema vs Local Extrema



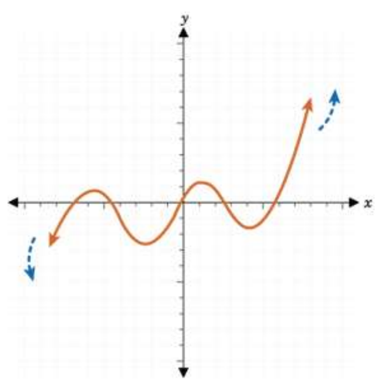
Global Maximum?
No, it goes up to infinity.

Global Minimum?
Yes, in Quadrant IV at about y= -3.

Local Maximums?
Yes, one in Quadrant II at about y= +1.

Local Minimums?
Yes, two.
One in Quadrant III at about y= -1.5.
Another in Quadrant IV at about y= -3.

Global Extrema vs Local Extrema



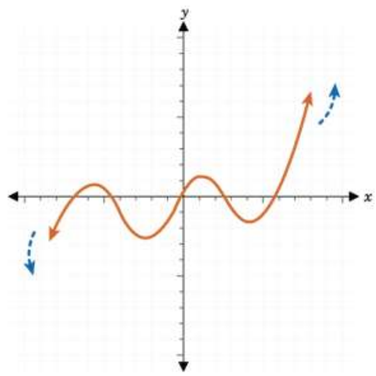
Global Maximum?

Global Minimum?

Local Maximums?
In which quadrants?

Local Minimums?
In which quadrants?

Global Extrema vs Local Extrema



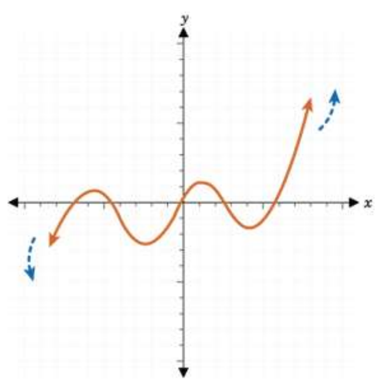
Global Maximum? No

Global Minimum? No

Local Maximums?
Yes, in Quadrants I and II.

Local Minimums?
Yes, in Quadrants III and IV.

Extreme Value Theorem



If you pick a closed interval, and the graph is continuous in that interval, then there will be a maximum and a minimum in that interval.

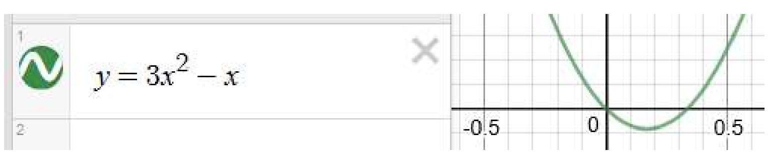
And, yes, the max or min might be the same as one, or both, of the endpoints of the interval.

That was a lot of new vocabulary . . .

The rest we saw last year in Algebra 2.

Zeros of Polynomials:

Where the function equals zero.
That is, where y = 0.
So, wherever the graph intersects the x-axis!



Degree 2 → 2 zeros **2 x-intercepts**

Zeros of Polynomials:

Remember, the **degree** of the polynomial tells you **how many zeros** it has!

… although, it may not show that many on the graph, as there can be multiplicities
… that is, you may get duplicate solutions

OR, there may be complex number solutions!!

Zeros of Polynomials:

For example, if the graph shows 2 x-intercepts, but the polynomial is degree 3 …

Then you would get one of those zero values twice when you solve for them algebraically in order to get a total of 3 zero values!!

Zeros of Polynomials:

But, if the graph shows 2 x-intercepts and the polynomial is degree 4 …

Then, the other two zero values could be duplicates, OR, a conjugate pair of complex numbers (remember the form a ± b*i*).

Zeros of Polynomials:

IF there are any complex, or irrational, zeros … they always come in conjugate pairs!

For example:
If 5+2i is a zero, then 5-2i must also be a zero.

Factors of Polynomials:

Remember that zeros tell you the factors. And factors tell you the zeros.

For example:
If the zeros are x = 2, 3, -5
Then the factors are (x-2)(x-3)(x+5)

Check: x-2=0 → x=2
x-3=0 → x=3
x+5=0 → x=-5

Dividing Polynomials:

Once you identify one x value that is a zero, you can divide the polynomial by the factor that goes with that zero.

Then check the resulting new polynomial for the next zero.

Dividing Polynomials:

If the graph shows an intercept at x = -2,
or if you are told that x = -2 is a zero of the polynomial,
then (x + 2) is a factor of the polynomial.

Check this by dividing the polynomial by x+2
using long division, or check it quicker
using synthetic division dividing by x=-2.

If the remainder is 0, then you have confirmed it is a zero of the polynomial.

Dividing Polynomials:

Next, divide:

$$\begin{array}{r} x^2+2x-3 \\ x+2\overline{\smash{\big)}\,x^3+4x^2+x-6} \\ \underline{x^3+2x^2\quad\quad\;\;} \\ 2x^2+x\quad\; \\ \underline{2x^2+4x\quad\;} \\ -3x-6 \\ \underline{-3x-6} \\ 0 \end{array}$$

Next, solve $x^2 + 2x + 3 = 0$ to get the next zeros.

Questions??

Review the Key Terms and Key Concepts documents for this unit.

Look up the topic at khanacademy.org and virtualnerd.com

Check our class website at nca-patterson.weebly.com

***Reserve a time for a call with me at**
jpattersonmath.youcanbook.me
We can use the LiveLesson whiteboard
to go over problems together.