UNIT 5 Lessons 1, 3

PRECALCULUS A

LESSONS:

- Exponential Functions & Graphs
- Graphs of Logarithmic Functions

(we'll talk about simplifying and solving the equations next week)

The Parent Exponential Function:

$$f(x) = b^x$$

Where b is a positive real number
and b is not equal to 1.

… it is an exponential function because the variable is in the exponent!

The Parent Exponential Function:

$$f(x) = b^x$$

Where b is a positive real number
and b is not equal to 1.

What would happen if b were equal to 1?

Special Cases:

Exponential Growth:
Where a > O and b > 1

$$f(x) = ab^x$$

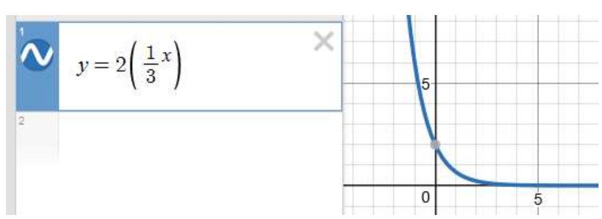
Exponential Decay:
Where a > O and O < b < 1

$$f(x) = ab^x$$

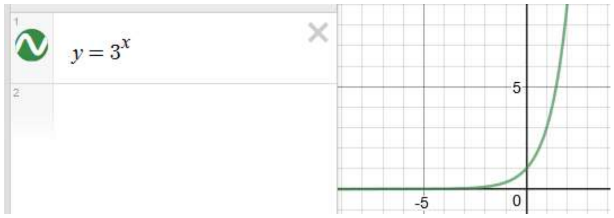
. . . FYI: a is the starting amount
And you don't start with a negative amount . . .

EXPONENTIAL GROWTH

EXPONENTIAL DECAY



EXPONENTIAL GROWTH



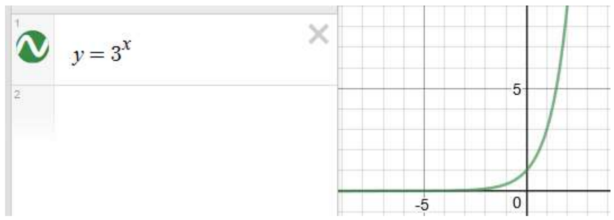
Notice that "a" is also the y-intercept

EXPONENTIAL DECAY

Let's do all of the features!

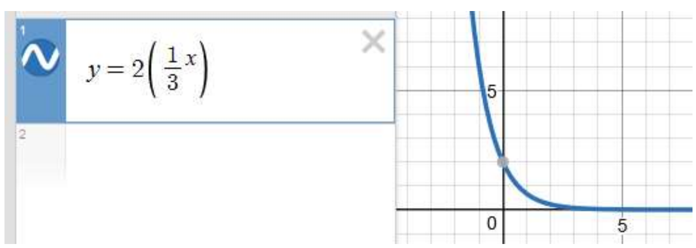
Domain:
Range:
X-intercept:
Y-intercept:
Asymptote:
End Behavior:

Let's do all of the features!



Domain: (-infinity, +infinity)
Range: (0, +infinity)
X-intercept: none
Y-intercept: (0, 1)
Asymptote: y = 0
End Behavior: f(x) → ∞ as x→ ∞,
f(x) → 0 as x → -∞

Let's do all of the features!



Domain:
Range:
X-intercept:
Y-intercept:
Asymptote:
End Behavior:

Let's do all of the features!

Domain: (-infinity, +infinity)
Range: (0, +infinity)
X-intercept: none
Y-intercept: (0, 2)
Asymptote: y = 0
End Behavior: f(x) → 0 as x → ∞,
f(x) → ∞ as x → -∞

Now for the Transformations!
. . . of course ☺

1st – Shifts (also known as Translations)

2nd – Stretch/Compress

3rd – Reflections

SHIFTS (TRANSLATIONS)

Vertical shifts (k) are added to the end of the function.

$$g(x) = b^x + k$$

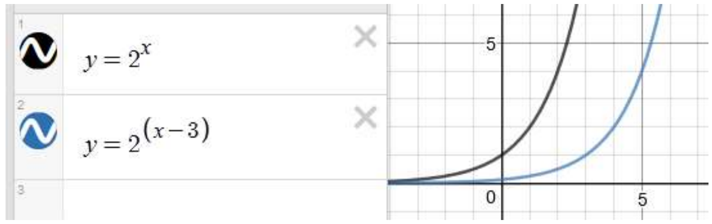
Horizontal shifts (h) are subtracted from the x before doing the parent function.

$$g(x) = b^{x-h}$$

Transformation: ___________

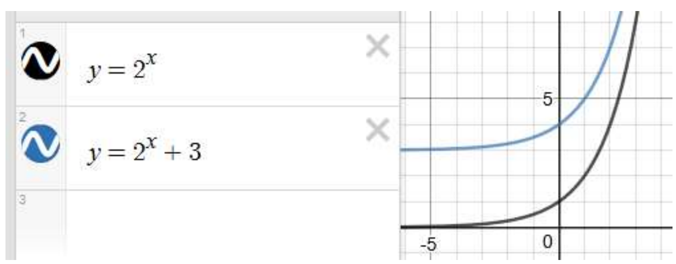
Same or Different:

- Domain
- Range
- X-intercept
- Y-intercept
- Asymptote
- End Behavior



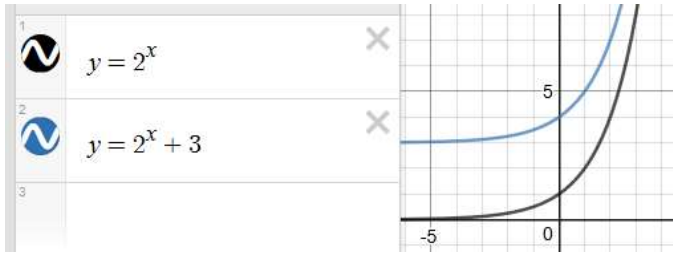
Transformation: Shift 3 right

Same or Different:
- Domain - same
- Range - same
- X-intercept - same (none)
- Y-intercept - different
- Asymptote - same
- End Behavior - same



Transformation: __________

Same or Different:
- Domain
- Range
- X-intercept
- Y-intercept
- Asymptote
- End Behavior



Transformation: Shift 3 up

Same or Different:

- Domain - same
- Range - different (bottom is now 3)
- X-intercept - same (none)
- Y-intercept - different
- Asymptote - different (now y = 3)
- End Behavior - different (now → 3, not 0)

STRETCH OR COMPRESS

Vertical stretches or compressions are multiplied at the beginning of the function.

$$g(x) = ab^x$$

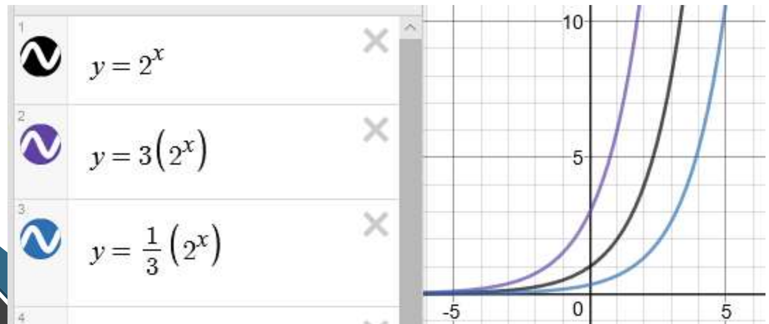
Horizontal stretches or compressions are multiplied by the x before doing the parent function.

$$g(x) = b^{cx}$$

$$g(x) = ab^x$$

A vertical stretch will be when a > 1
the graph stretches closer to the vertical axis
by a factor of a

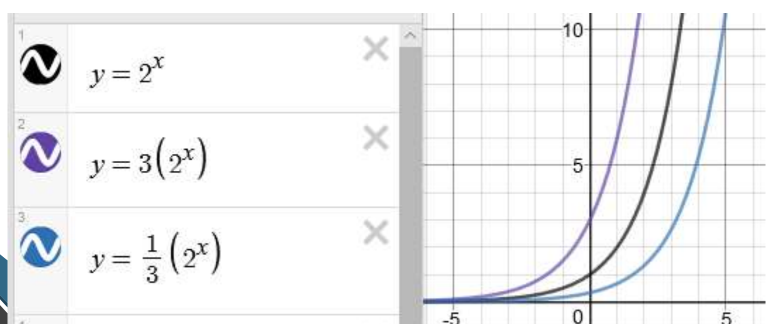
A vertical compression is when 0 < a < 1
the graph compresses away from the vertical axis
by a factor of a



VERTICAL STRETCH/COMPRESS $g(x) = ab^x$

Same or Different:

- Domain
- Range
- X-intercept
- Y-intercept
- Asymptote
- End Behavior

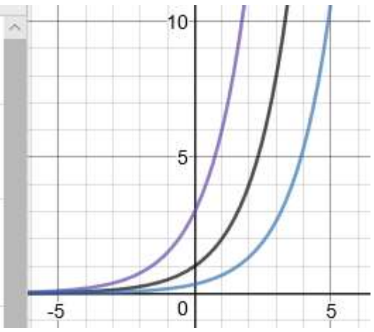


VERTICAL STRETCH/COMPRESS $g(x) = ab^x$

Same or Different:

- Domain – same
- Range – same
- X-intercept – same (none)
- Y-intercept – different
- Asymptote – same
- End Behavior – same

1. $y = 2^x$
2. $y = 3\left(2^x\right)$
3. $y = \frac{1}{3}\left(2^x\right)$

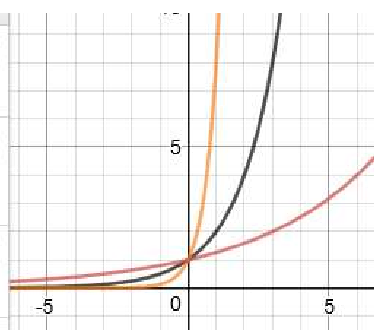


$g(x) = b^{cx}$

A <u>horizontal stretch</u> will be when 0 < c < 1
the graph stretches closer to the horizontal axis
by a factor of 1/c

A <u>horizontal compression</u> will be when c > 1
the graph compresses away from the horizontal axis
by a factor of 1/c

1. $y = 2^x$
2. $y = 2^{\frac{1}{3}x}$
3. $y = 2^{3x}$

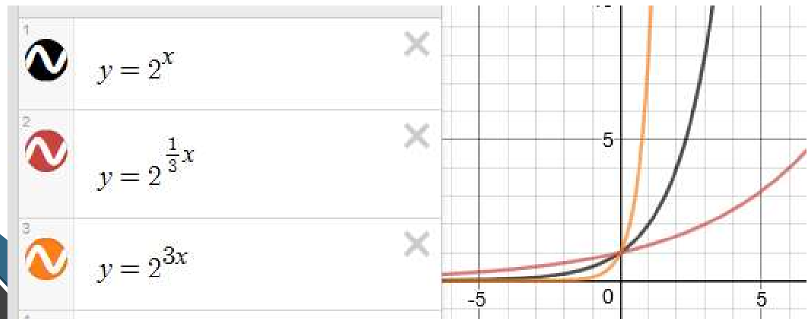


HORIZONTAL STRETCH/COMPRESS

$$g(x) = b^{cx}$$

Same or Different:

- Domain
- Range
- X-intercept
- Y-intercept
- Asymptote
- End Behavior

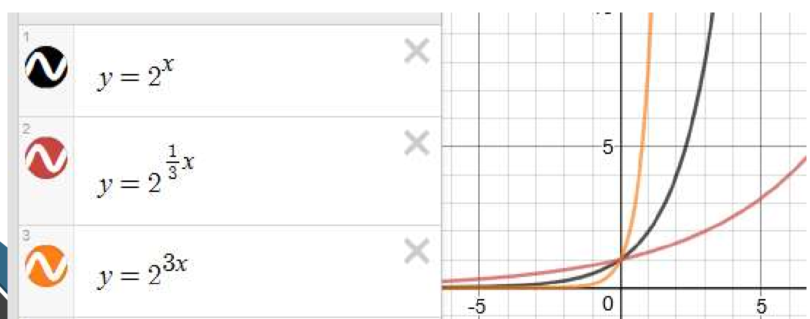


HORIZONTAL STRETCH/COMPRESS

$$g(x) = b^{cx}$$

Same or Different:

- Domain – same
- Range – same
- X-intercept – same (none)
- Y-intercept – same
- Asymptote – same
- End Behavior – same



REFLECTION

A vertical reflection over the x-axis happens when the negative is at the beginning of the function

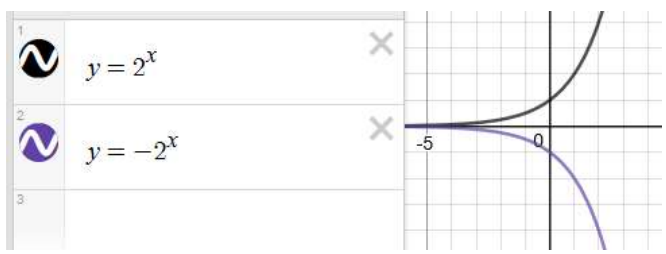
$$g(x) = -b^x$$

A horizontal reflection over the y-axis happens when the negative is on the x before doing the parent function.

$$g(x) = b^{-x}$$

VERTICAL REFLECTION

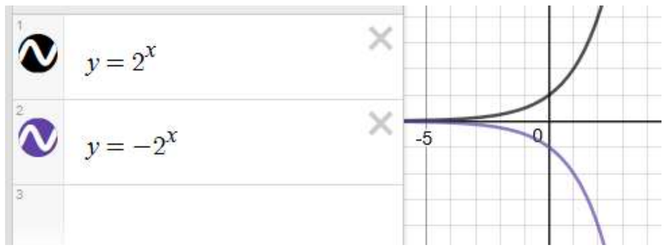
$$g(x) = -b^x$$



Same or Different:

- Domain
- Range
- X-intercept
- Y-intercept
- Asymptote
- End Behavior

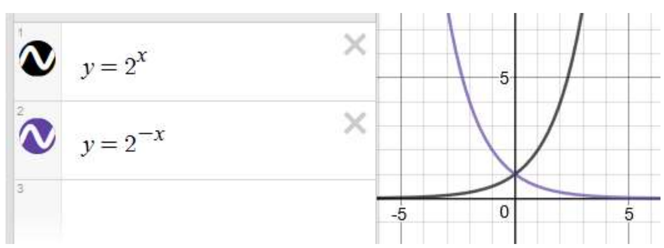
VERTICAL REFLECTION $g(x) = -b^x$



Same or Different:

- Domain – same
- Range – different (reverses)
- X-intercept – same (none)
- Y-intercept – different (opposite)
- Asymptote – same
- End Behavior – different (in the x → +∞ direction)

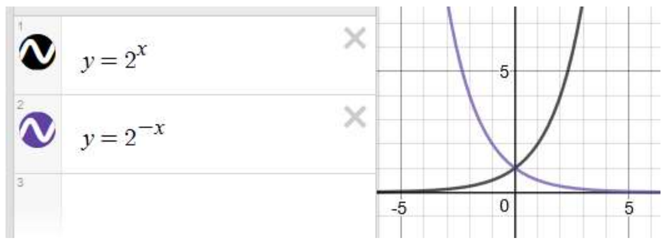
HORIZONTAL REFLECTION $g(x) = b^{-x}$



Same or Different:

- Domain
- Range
- X-intercept
- Y-intercept
- Asymptote
- End Behavior

HORIZONTAL REFLECTION $g(x) = b^{-x}$



Same or Different:

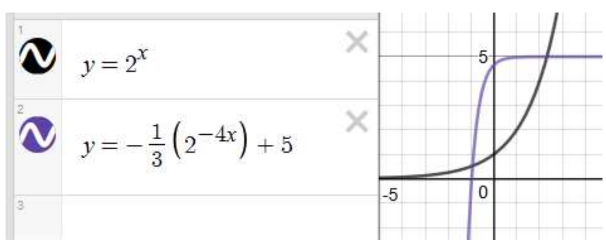
- Domain – same
- Range – same
- X-intercept – same (none)
- Y-intercept – same
- Asymptote – same
- End Behavior – different (reverses)

Now to do them all together . . .

$$f(x) = ab^{c(x-h)} + k$$

Where a and c are not equal to zero,
and b is a positive real number not equal to 1

Try changing this and see what happens!



b = 2 a = 1/3 c = 4 h = 0 k = 5
No Horizontal shift
Vertical compression by factor of 1/3
Horizontal compression by factor of ¼
Vertical and Horizontal reflections
Vertical shift 5 up

NEXT: the Inverse!!

The inverse of the EXPONENTIAL FUNCTION

$$f(x) = b^x$$

is the LOGARITHMIC FUNCTION

$$f(x) = \log_b x$$

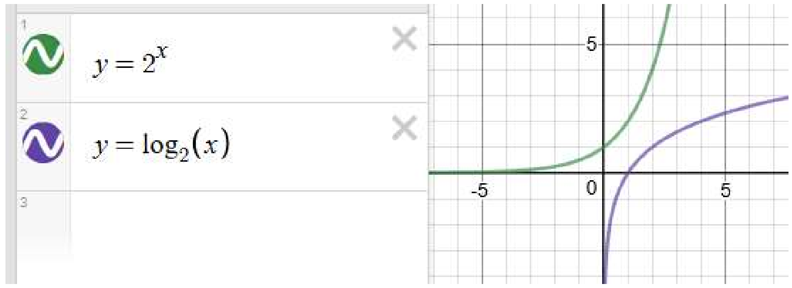
REMINDER:

Logarithms equal the exponent, and have the same base as the exponential function.

Exponential Form: $2^x = 16$

Logarithmic Form: $\log_2 16 = x$

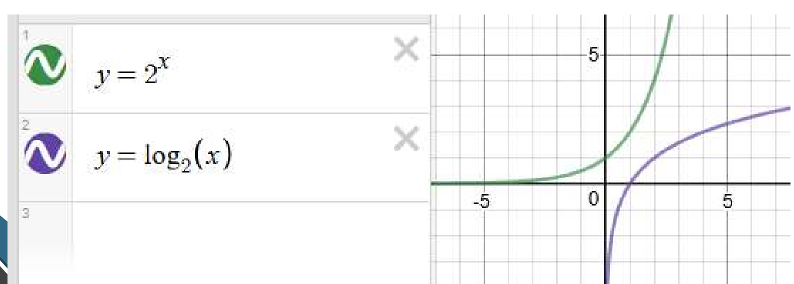
. . . we did this last year, and we will do more with these equations next week.



INVERSE FUNCTIONS:

- **Are reflections across the diagonal line y=x**
- **Compositions reverse back to the starting amount**
- **Switch their domain and range**

	Exponential Function	**Logarithmic Function**
function form	$f(x) = b^x$	$f(x) = \log_b x$
domain	$(-\infty, \infty)$	$(0, \infty)$
range	$(0, \infty)$	$(-\infty, \infty)$
asymptote	x-axis	y-axis
x-intercept	none	$(1, 0)$
y-intercept	$(0, 1)$	none



And, yes, we look at transformations of logarithms as well . . .

SHIFTS (TRANSLATIONS)

Vertical Shift $g(x) = \log_b(x) + k$

Horizontal Shift $g(x) = \log_b(x - h)$

STRETCH OR COMPRESS

Vertical stretches or compressions are multiplied at the beginning of the function.
When a > 1,
the graph stretches closer to the vertical axis by a factor of a
When 0 < a < 1,
the graph compresses away from the vertical axis by a factor of a

$$g(x) = a\log_b x$$

Horizontal stretches or compressions are multiplied by the x before doing the parent function.
When 0 < c < 1,
the graph stretches closer to the horizontal axis by a factor of 1/c
When c > 1,
the graph compresses away from the horizontal axis by a factor of 1/c

$$g(x) = \log_b(cx)$$

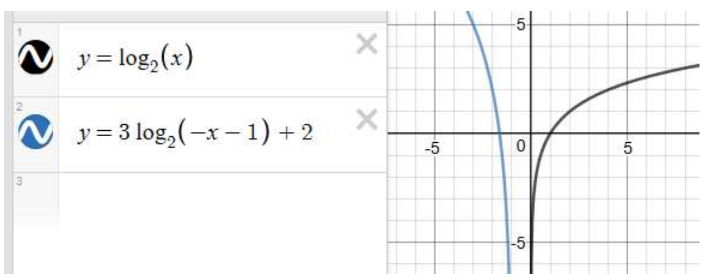
REFLECTION

A vertical reflection over the x-axis happens when the negative is at the beginning of the function

$$g(x) = -\log_b x$$

A horizontal reflection over the y-axis happens when the negative is on the x before doing the parent function.

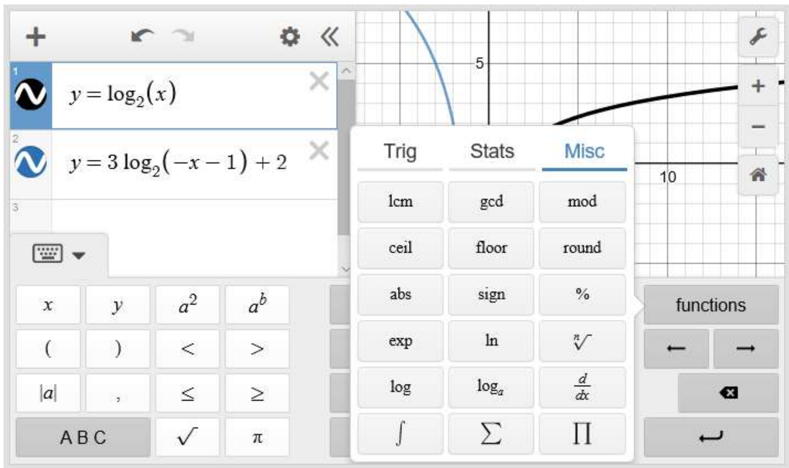
$$g(x) = \log_b(-x)$$



All together:

- Horizontal shift of 1 right
- Vertical stretch by a factor of 3
- Reflection over the y-axis
- Vertical shift of 2 up

By the way, to get a log in Desmos . . . Go to the Functions button and select the Misc tab.



Questions??

Review the Key Terms and Key Concepts documents for this unit.

Look up the topic at khanacademy.org and virtualnerd.com

Check our class website at nca-patterson.weebly.com

*Reserve a time for a call with me at
jpattersonmath.youcanbook.me
We can use the LiveLesson whiteboard
to go over problems together.