# UNIT 7 LESSONS 1-5

PRECALCULUS A

## ARITHMETIC SEQUENCES & SERIES

## SEQUENCES

A sequence is a list of numbers that follows a consistent pattern.

The pattern can be written as a rule or formula.

## NOTATION FOR SEQUENCES

- $a_n$ is the term in position $n$, or the "$n^{th}$ " term
- $a_1$ is the 1st term, $a_2$ is the 2nd term, and so forth
- $a_{n-1}$ is the previous term, $a_{n+1}$ is the next term
- $n$ is the term number, or, the position in the list

  (that is 1 for 1st , 2 for 2nd, …)

## ARITHMETIC SEQUENCE

Definition: an ordered list of numbers with a constant difference, that is, the same number gets added to each term to get the next term on the list.

*d* is the amount of the constant difference

For example, odd numbers have a constant difference of 2.

## RULES: RECURSIVE & EXPLICIT

A **Recursive Rule** describes the sequence by telling what you do to the term before to get the next term. It must also give the first number of the sequence as a starting point.

An **Explicit Rule** describes the sequence with what you do to the starting term directly based on which term position you need.

## RECURSIVE RULE for an ARITHMETIC SEQUENCE

$a_n = a_{n-1} + d; a_1 = \text{a}$

For example, the sequence 1, 3, 5, 7, … has a constant difference of 2 and the first term is the number 1, so it's recursive rule would be:

$a_n = a_{n-1} + 2; a_1 = 1$

## RECURSIVE RULE for an ARITHMETIC SEQUENCE

For example, $a_n = a_{n-1} + 2; a_1 = 1$

In other words … "any term on this list is equal to the previous term plus 2, and we started at 1".

## EXPLICIT RULE for an ARITHMETIC SEQUENCE

$$a_n = a_1 + (n-1)d$$

For example, the same sequence 1, 3, 5, 7, … has the explicit rule of: $a_n = 1 + (n-1)2$

After distributing and simplifying, this rule can also be written as: $a_n = 2n - 1$

## EXPLICIT RULE for an ARITHMETIC SEQUENCE

For example, $a_n = 1 + (n-1)2$

In other words … any term on this list is equal to the starting number 1 + 2 times 1 less than the position number of the desired term.

Why n-1? Because, for example, for the 10th term on the list, you've added the difference 9 times.

## EXPLICIT RULE for an ARITHMETIC SEQUENCE

Let's look at a few example terms for $a_n$ = 1 + (n-1)2

- 1st term: $a_1$ = 1 + (1-1)2 = 1 + (0)2 = 1
- 2nd term: $a_2$ = 1 + (2-1)2 = 1 + (1)2 = 3 … added 2 once
- 3rd term: $a_3$ = 1 + (3-1)2 = 1 + (2)2 = 5 … added 2 twice
- 10th term: $a_{10}$ = 1 + (10-2)2 = 1 + (9)2 = 19

  … added 2 nine times

## ARITHMETIC MEAN

This is for finding the number between two terms of an Arithmetic Sequence.

$$\frac{x+y}{2}$$

For example, the number between 3 and 7 in the previous example sequence above is $\frac{3+7}{2} = 5$.

## SERIES

A Series is the sum of the terms of a Sequence.

So, essentially, just replace the commas with addition signs.

## TYPES OF SERIES

**Finite series** have a limited number of terms.
For example, 1 + 3 + 5 + 7 is a finite series.

**Infinite series** have an endless number of terms.
For example, 1 + 3 + 5 + 7 + 9 + … is an infinite series.

## SIGMA NOTATION

This is the shorthand way for writing a series by giving the pattern for a sequence and which terms are being added.

Sigma ($\sum$) is the Greek letter for capital S, and is used to mean a sum. This notation can be used for describing any type of sequence that is being added as a series.

## SIGMA NOTATION

Below the $\sum$ is written the n value for the first position to be added in the series.

Above the $\sum$ is written the n value for the last position to be added in the series.

To the right of the $\sum$ is written the Explicit rule for the sequence being used in the summation.

## SIGMA NOTATION

For example, this is the summation notation for adding the first 5 odd numbers.

$$\sum_{n=1}^{5} 2n - 1$$

Notice that the Explicit Rule is written in the version you get after distributing and simplifying $a_n = 1 + (n-1)2$.

## SIGMA NOTATION

$$\sum_{n=1}^{5} 2n - 1$$

In other words … this means to start with plugging in 1 for n into 2n-1 to get the first term of the sequence. Then plug in 2 for n to get the second term, and so forth to get all five terms indicated. Then add these five terms to get the sum of this finite series.

## NOTATION FOR SERIES

- $S_n$ is the sum of the first n terms, and is called the nth partial sum
- $n$ is the number of terms to be added
- $a_1$ is the first number in the series
- $a_n$ is the last number to be added in the series

## SUM of a FINITE ARITHMETIC SERIES

The Arithmetic formula is: $S_n = \frac{n}{2}(a_1 + a_n)$

For example, the sum of the first five odd numbers is:

$$S_5 = \frac{5}{2}(1 + 9) = 25$$

because the first number being added is 1, the last number being added is 9, and there are 5 numbers being added.

## SUM of an INFINITE ARITHMETIC SERIES

This has an undefined result.

Adding a list that has no end and each number is larger than the last has an ever-growing sum.

We call this a Divergent Series, as it never converges on a defined amount.

## Questions??

Review the Key Terms and Key Concepts documents for this unit.

Look up the topic at khanacademy.org and virtualnerd.com

Check our class website at nca-patterson.weebly.com

*Reserve a time for a call with me at
jpattersonmath.youcanbook.me
We can use the LiveLesson whiteboard
to go over problems together.