UNIT 7 LESSONS 6-10

PRECALCULUS A

GEOMETRIC SEQUENCES & SERIES

SEQUENCES

A sequence is a list of numbers that follows a consistent pattern.

The pattern can be written as a rule or formula.

NOTATION FOR SEQUENCES

- a_n is the term in position n, or the "n^{th} " term
- a_1 is the 1st term, a_2 is the 2nd term, and so forth
- a_{n-1} is the previous term, a_{n+1} is the next term
- *n* is the term number, or, the position in the list (that is 1 for 1st , 2 for 2nd, …)

GEOMETRIC SEQUENCE

Definition: an ordered list of numbers with a constant **ratio**, that is, the same number gets **multiplied** to each term to get the next term on the list.

r is the amount of the constant ratio

For example: 2, 6, 18, 54, 162, … has a common ratio of 3.

RULES: RECURSIVE & EXPLICIT

A **Recursive Rule** describes the sequence by telling what you do to the term before to get the next term. It must also give the first number of the sequence as a starting point.

An **Explicit Rule** describes the sequence with what you do to the starting term directly based on which term position you need.

RECURSIVE RULE for a GEOMETRIC SEQUENCE

$$a_n = a_{n-1} \cdot r \,;\, a_1 = \text{a}$$

For our example of 2, 6, 18, 54, 162, … with a common ratio of 3, the rule is: $a_n = a_{n-1} \cdot 3 \,;\, a_1 = 2$

In other words … any term on this list is equal to the previous term times 3, and we started at 2.

EXPLICIT RULE for a GEOMETRIC SEQUENCE

$$a_n = a_1 \cdot r^{n-1}$$

So our example of 2, 6, 18, 54, 162, … with a common ratio of 3, the rule is: $a_n = 2 \cdot 3^{n-1}$

In other words, any term on this list is equal to the first term times the ratio 1 time less than the position number we want.

EXPLICIT RULE for a GEOMETRIC SEQUENCE

Why n-1? Because, for example, for the 10th term on the list, you've multiplied by the ratio 9 times.

You don't multiply to get the first number.

GEOMETRIC MEAN

This is for finding the number between two terms of a Geometric Sequence.

$$\sqrt{xy}$$

For example, the number between 2 and 18 in the previous example sequence is $\sqrt{2 \cdot 18} = \sqrt{36} = 6$.

SERIES

A Series is the sum of the terms of a Sequence.

So, essentially, just replace the commas with addition signs.

TYPES OF SERIES

Finite series have a limited number of terms.
For example, 2 + 6 + 18 + 54 is a finite geometric series.

Infinite series have an endless number of terms.
For example, 2 + 6 + 18 + … is an infinite geometric series.

SIGMA NOTATION

This is the shorthand way for writing a series.

Below the $\sum$ is written the n value for the first position to be added in the series.

Above the $\sum$ is written the n value for the last position to be added in the series.

To the right of the $\sum$ is written the Explicit rule for the sequence being used in the summation.

NOTATION FOR SERIES

- S_n is the sum of the first n terms, and is called the nth partial sum
- n is the number of terms to be added
- a_1 is the first number in the series
- a_n is the last number to be added in the series

SUM of a FINITE GEOMETRIC SERIES

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

For our example, the sum of 2 +6 +18 +54 is found by:

$$S_4 = \frac{2(1-3^4)}{1-3} = \frac{2(1-81)}{-2} = 80$$

because the first number being added is 2, the last number being added is 54, and there are 4 numbers being added.

SUM of an INFINITE GEOMETRIC SERIES

This has an **undefined result if the absolute value of the ratio is greater than 1.**

Adding a list that has no end and each number is larger than the last has an ever-growing sum.

We call this a **Divergent Series**, as it never converges on a defined amount.

SUM of an INFINITE GEOMETRIC SERIES

However, … this does have a **defined result if the absolute value of the ratio is less than 1!**

We call this a **Convergent Series**, as it approaches a defined amount.

For example, $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$ which has a constant ratio of ½, is a convergent infinite geometric series.

SUM of an INFINITE GEOMETRIC SERIES

For a geometric series like this where the absolute value of the ratio is less than 1,

… although we keep adding another amount, those amounts eventually get so small, it is almost as if we are adding nothing!

Hence, we say it "converges" on a defined sum.

SUM of an INFINITE GEOMETRIC SERIES

$$S = \frac{a_1}{1-r}; \text{ for } |r| < 1$$

For our example, $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$ with a ratio of ½,

$$S = \frac{1}{1-\frac{1}{2}} = 2$$

Which means the infinites sum converges on the value of 2.

Questions??

Review the Key Terms and Key Concepts documents for this unit.

Look up the topic at khanacademy.org and virtualnerd.com

Check our class website at nca-patterson.weebly.com

*Reserve a time for a call with me at
jpattersonmath.youcanbook.me
We can use the LiveLesson whiteboard
to go over problems together.