Function Algebra Key Concepts

Domain and Range of a Function Lesson

Interval Notation

The open interval (a, b) is equivalent to the set $\{x \mid a < x < b\}$.

The closed interval $[a, b]$ is equivalent to the set $\{x|a \leq x \leq b\}$.

The half-open interval $(a, b]$ is equivalent to the set $\{x|a < x \leq b\}$.

The half-open interval $[a, b)$ is equivalent to the set $\{x|a \leq x < b\}$.

The non-ending interval $(-\infty, b)$ is equivalent to the set $\{x|x < b\}$.

The non-ending interval $[a, \infty)$ is equivalent to the set $\{x|x \geq a\}$.

The non-ending interval $(-\infty, \infty)$ represents all real numbers, which is also written as $\{x|x \in \mathbb{R}\}$.

Algebra of Functions Lesson

sum of functions	$(f + g)(x) = f(x) + g(x)$ for functions *f* and *g*
difference of functions	$(f - g)(xx) = f(x) - g(x)$ for functions *f* and *g*

product of functions	$(f \cdot g)(x) = f(x) \cdot g(x)$ for functions *f* and *g*
quotient of functions	$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ for functions *f* and *g* where $g(x) \neq 0$

Composition of Functions Lesson

Composition of Functions

For two functions *f* and *g*, the composition of *f* and *g* is $f \circ g$, where $f \circ g(x) = f\big(g(x)\big)$.

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Inverse Functions Lesson

One-to-One Function

A function, *f*,is one-to-one if $f(x_1) = f(x_2)$ implies $x_1 = x_2$.

Inverse Functions

- A function, *f*, has an inverse function if it is one-to-one.
- For each ordered pair of points from the function *f*, interchanging the corresponding domain and range values results in another function, called the inverse of $f(x)$.
- The inverse of *f* is denoted by the notation f^{-1}.
- Note: The -1 in the inverse function notation is **not** an exponent. This means that f^{-1} does **not** represent the reciprocal function, $f^{-1}(x) \neq \frac{1}{f(x)}$.

Domain and Range of a Function and its Inverse

For a function, *f*, and its inverse, f^{-1}, the domain of *f* is the range of f^{-1}, and the domain f^{-1} of is the range of *f*.

Steps for Finding the Inverse of a Function

1. Replace $f(x)$ with *y*.
2. Interchange *x* and *y.*
3. Solve for *y*.
4. Replace *y* with $f^{-1}(x)$.

Verifying Inverse Functions Lesson

Composing Inverse Functions

If *f* and *g* are inverse functions, then $f\big(g(x)\big) = x$ for all *x*-values in the domain of *g*, and $g\big(f(x)\big) = x$ for all *x*-values in the domain of *f*.

Steps to Prove that *f* and *g* are Inverse Functions

1. Show that $f\big(g(x)\big) = x$.
2. Show that $g\big(f(x)\big) = x$.

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Graphs of Inverse Functions Lesson

Properties of Graphs of Inverse Functions

Graphs of a one-to-one function, *f*, and its inverse function, f^{-1}, have the following three properties:

- The graphical representations for *f* and f^{-1} are symmetrical about the line *y* = *x*.
- All points of intersection for the graphs of *f* and f^{-1} are located along the line *y* = *x*.
- If the point (a, b) is on the graph of the function *f*, then the point (b, a) is on the graph of the function f^{-1}.

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