Graph Behavior Key Concepts

Analyzing Functions Lesson

Increasing, Decreasing, and Constant Intervals

- A function is increasing on an open interval in which $f(x_1) < f(x_2)$ when $x_1 < x_2$ for all x_1 and x_2 in the interval.
- A function is decreasing on an open interval in which $f(x_1) > f(x_2)$ when $x_1 < x_2$ for all x_1 and x_2 in the interval
- A function is constant on an open interval in which $f(x_1) = f(x_2)$ for all x_1 and x_2 in the interval.

Even and Odd Functions Lesson

Even and Odd Functions

- A function *f* is even if for each value of *x* in the domain, $f(-x) = f(x)$. The graph of an even function displays symmetry with respect to the *y*-axis; if the point (x, y) lies on the graph of *f*, then the point $(-x, y)$ also lies on the graph of *f*.
- A function *f* is odd if for each value of *x* in the domain, $f(-x) = -f(x)$. The graph of an odd function displays symmetry with respect to the origin; if the point (x, y) lies on the graph of *f*, then the point $(-x, -y)$ also lies on the graph of *f*.

Test for Even or Odd Functions

To test if a function *f* is even, odd, or neither, substitute $-x$ for *x* and simplify.

- If $f(-x) = f(x)$, then the function is even.
- If $f(-x) = -f(x)$, then the function is odd.
- Otherwise, the function is neither even nor odd.

Asymptotes and End Behavior Lesson

Vertical and Horizontal Asymptotes

The line $x = a$ is a vertical asymptote of the graph of $f(x)$ if $f\left(x\right) \to \infty$ or $f\left(x\right) \to -\infty$ as $x \to a$.

The line $y = b$ is a horizontal asymptote of the graph of $\boldsymbol{f(x)}$ if $f\left(x\right) \to b$ as $x \to \infty$ or $x \to -\infty$.

Continuous and Discontinuous Functions Lesson

Continuous and Discontinuous Functions

A function is continuous if its graph is a single, unbroken curve.

A function is discontinuous if its graph has a hole, jump, or vertical asymptote.

Linear, Absolute Value, and Reciprocal Functions Lesson

Characteristics of the Parent Linear Function: $f(x) = x$

- domain: $(-\infty, \infty)$
- range: $(-\infty, \infty)$
- increasing intervals: $(-\infty, \infty)$
- decreasing intervals: none
- constant intervals: none
- x-intercept: $(0, 0)$
- y-intercept: $(0, 0)$
- even, odd, neither: odd
- continuous or discontinuous: continuous
- asymptotes: none
- end behavior: $f(x) \to \infty \text{ as } x \to \infty$; $f(x) \to -\infty \text{ as } x \to -\infty$

Characteristics of the Constant Function: $f(x) = c$

- domain: $(-\infty, \infty)$
- range: *c*
- increasing intervals: none
- decreasing intervals: none
- constant intervals: $(-\infty, \infty)$
- *x*-intercept: none if $c \neq 0$; if *c* = 0, the function's graph is the *x*-axis
- *y*-intercept: $(0, c)$
- even, odd, neither: even; if *c* = 0 the function is also odd
- continuous or discontinuous: continuous
- asymptotes: none
- end behavior: $f(x) \to c$ as $x \to \infty$; $f(x) \to c$ as $x \to -\infty$

Characteristics of the Parent Absolute Value Function: $f(x) = |x|$

- domain: $(-\infty, \infty)$
- range: $[0, \infty)$
- increasing intervals: $(0, \infty)$
- decreasing intervals: $(-\infty, 0)$
- constant intervals: none
- *x*-intercept: $(0, 0)$
- *y*-intercept: $(0, 0)$
- even, odd, neither: even
- continuous or discontinuous: continuous
- asymptotes: none
- end behavior: $f(x) \to \infty$ as $x \to \infty$; $f(x) \to \infty$ as $x \to -\infty$

Characteristics of the Parent Reciprocal Function:

$f(x)=\frac{1}{x}$

- domain: $(-\infty,0)\cup(0,\infty)$
- range: $(-\infty,0)\cup(0,\infty)$
- increasing intervals: none
- decreasing intervals: $(-\infty,0)\cup(0,\infty)$
- constant intervals: none
- x-intercept: none
- y-intercept: none
- even, odd, neither: odd
- continuous or discontinuous: discontinuous
- asymptotes: $x = 0$ and $y = 0$
- end behavior: $f(x)\to 0 \text{ as } x\to\infty$; $f(x)\to 0 \text{ as } x\to-\infty$

Power, Root, Exponential, Logarithmic Functions Lesson

Characteristics of Even-Degree Power Functions: $f(x)=x^n$, where *n* is an even integer greater than zero

- domain: $(-\infty,\infty)$
- range: $[0,\infty)$
- increasing intervals: $(0,\infty)$
- decreasing Intervals: $(-\infty,0)$
- constant intervals: none
- x-intercept: $(0,0)$
- y-intercept: $(0,0)$
- even, odd, neither: even
- continuous or discontinuous: continuous
- asymptotes: none
- end behavior: $f(x)\to\infty \text{ as } x\to\infty$; $f(x)\to\infty \text{ as } x\to-\infty$

Characteristics of Odd-Degree Power Functions:

$f(x)=x^n$, where *n* is an odd integer greater than zero

- domain: $(-\infty,\infty)$
- range: $(-\infty,\infty)$
- increasing Intervals: $(-\infty,\infty)$
- decreasing intervals: none
- constant intervals: none
- *x*-intercept: $(0,0)$
- *y*-intercept: $(0,0)$
- even, odd, neither: odd
- continuous or discontinuous: continuous
- asymptotes: none
- end behavior: $f(x)\to\infty$ as $x\to\infty$; $f(x)\to-\infty$ as $x\to-\infty$

Characteristics of Even-Index Root Functions,

$f(x)=\sqrt[n]{x}$, where *n* is an even integer greater than zero

- domain: $[0,\infty)$
- range: $[0,\infty)$
- increasing intervals: $(0,\infty)$
- decreasing intervals: none
- constant intervals: none
- *x*-intercept: $(0,0)$
- *y*-intercept: $(0,0)$
- even, odd, neither: neither
- continuous or discontinuous: continuous
- asymptotes: none
- end behavior: $f(x)\to\infty$ as $x\to\infty$

Characteristics of Odd-Index Root Functions:
$f(x)=\sqrt[n]{x}$, where *n* is an odd integer greater than zero

- domain: $(-\infty,\infty)$
- range: $(-\infty,\infty)$
- increasing intervals: $(-\infty,\infty)$
- decreasing intervals: none
- constant intervals: none
- *x*-intercept: $(0,0)$
- *y*-intercept: $(0,0)$
- even, odd, neither: odd
- continuous or discontinuous: continuous
- asymptotes: none
- end behavior: $f(x)\to\infty \text{ as } x\to\infty$; $f(x)\to-\infty \text{ as } x\to-\infty$

Characteristics of Exponential Functions:

$f(x)=b^x$, $b>1$

- domain: $(-\infty,\infty)$
- range: $(0,\infty)$
- increasing intervals: $(-\infty,\infty)$
- decreasing intervals: none
- constant intervals: none
- *x*-intercept: none
- *y*-intercept: $(0,1)$
- even, odd, neither: neither
- continuous or discontinuous: continuous
- asymptotes: $y = 0$
- end behavior: $f(x)\to\infty \text{ as } x\to\infty$; $f(x)\to 0 \text{ as } x\to-\infty$

Characteristics of Exponential Functions:

$f(x) = b^x$, $0 < b < 1$

- domain: $(-\infty, \infty)$
- range: $(0, \infty)$
- increasing intervals: none
- decreasing intervals: $(-\infty, \infty)$
- constant intervals: none
- *x*-intercept: none
- *y*-intercept: $(0,1)$
- even, odd, neither: neither
- continuous or discontinuous: continuous
- asymptotes: $y = 0$
- end behavior: $f(x) \to 0$ as $x \to \infty$; $f(x) \to \infty$ as $x \to -\infty$

Characteristics of the Parent Logarithmic Function:

$f(x) = \log_b x$, $b > 1$

- domain: $(0, \infty)$
- range: $(-\infty, \infty)$
- increasing intervals: $(0, \infty)$
- decreasing intervals: none
- constant intervals: none
- *x*-intercept: $(1,0)$
- *y*-intercept: none
- even, odd, neither: neither
- continuous or discontinuous: continuous
- asymptotes: $x = 0$
- end behavior: $f(x) \to \infty$ as $x \to \infty$; $f(x) \to -\infty$ as $x \to 0$

Characteristics of the Parent Logarithmic Function:

$f(x) = \log_b x$, $0 < b < 1$

- domain: $(0, \infty)$
- range: $(-\infty, \infty)$
- increasing intervals: none
- decreasing intervals: $(0, \infty)$
- constant intervals: none
- *x*-intercept: $(1, 0)$
- *y*-intercept: none
- even, odd, neither: neither
- continuous or discontinuous: continuous
- asymptotes: *x* = 0
- end behavior: $f(x) \to -\infty$ as $x \to \infty$; $f(x) \to \infty$ as $x \to 0$

Transformations of Functions Lesson

Vertical and Horizontal Shifts

The graph of $y = f(x) + a$ is a vertical shift of the graph of $f(x)$.

- If *a* is positive, the graph shifts up *a* units.
- If *a* is negative, the graph shifts down *a* units.

The graph of $y = f(x + a)$ is a horizontal shift of the graph of $f(x)$.

- If *a* is positive, the graph shifts left *a* units.
- If *a* is negative, the graph shifts right *a* units.

Vertical and Horizontal Stretches and Compressions

The graph of $y = af(x)$ is a vertical stretch or compression of the graph of $f(x)$.

- If $|a| > 1$, the graph is stretched away from the *x*-axis.
- If $0 < |a| < 1$, the graph is compressed toward the *x*-axis.

The graph of $y = f(ax)$ is a horizontal stretch or compression of the graph of $f(x)$.

- If $|a| > 1$, the graph is compressed towards the *y*-axis.
- If $0 < |a| < 1$, the graph is stretched away from the *y*-axis.

Reflection Across the *x*-Axis and *y*-Axis

The graph of $y = -f(x)$ is a reflection across the *x*-axis of the graph of $f(x)$.

The graph of $y = f(-x)$ is a reflection across the *y*-axis of the graph of $f(x)$.

Absolute Value of a Function

The graph of $y = |f(x)|$ is given by $y = \begin{cases} f(x) & \text{if } f(x) \geq 0 \\ -f(x) & \text{if } f(x) < 0 \end{cases}$.

For any values of *x* for which $f(x) \geq 0$, the graph of $y = |f(x)|$ is the same as the graph of $f(x)$. For values of *x* for which $f(x) < 0$, the graph of $y = |f(x)|$ is a reflection of the graph of $f(x)$ across the *x*-axis.

Multiple Transformations of Functions Lesson

Order of Transformations of Functions

When carrying out multiple transformations of a function, perform them in the following order:

1. horizontal shift
2. stretch or compression
3. reflection
4. vertical shift