Exponential, Logarithmic, and Piecewise Functions Key Concepts

Exponential Functions and Graphs Lesson

Exponential Growth and Decay

- Exponential growth is modeled by functions of the form $f(x) = ab^x$, where $a > 0$ and $b > 1$.
- Exponential decay is modeled by functions of the form $f(x) = ab^x$, where $a > 0$ and $0 < b < 1$.

General Form of an Exponential Function

The general form of an exponential function is $f(x) = ab^{c(x-h)} + k$, where $a \neq 0$, b is a positive real number not equal to 1, and $c \neq 0$.

- If $b > 1$, the function is an increasing function.
- If $0 < b < 1$, the function is a decreasing function.
- If $a > 0$, the domain is $(-\infty, \infty)$ and the range is $(0, \infty)$.
- If $a < 0$, the domain is $(-\infty, \infty)$ and the range is $(-\infty, 0)$.

Expansion of *e*

$$e = \frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \cdots$$

$$e \approx 2.7182818284\ldots$$

Vertical and Horizontal Translations of Exponential Functions

Consider the parent exponential function $f(x)=b^x$, where b is a positive real number not equal to 1.

Function	Vertical and Horizontal Translations
$g(x)=b^{x+h}$, for $h>0$	translation of the graph of the function $f(x)$ h units to the left
$g(x)=b^{x-h}$, for $h>0$	translation of the graph of the function $f(x)$ h units to the right
$g(x)=b^x+k$, for $k>0$	translation of the graph of the function $f(x)$ k units up
$g(x)=b^x-k$, for $k>0$	translation of the graph of the function $f(x)$ k units down

Vertical and Horizontal Stretches and Compressions of Exponential Functions

Consider the parent exponential function $f(x)=b^x$, where b is a positive real number not equal to 1.

Function	Vertical and Horizontal Stretches and Compressions
$g(x)=ab^x$, for $a>1$	vertical stretch of the graph of the function $f(x)$ by a factor of a
$g(x)=ab^x$, for $0<a<1$	vertical compression of the graph of the function $f(x)$ by a factor of a
$g(x)=b^{cx}$, for $c>1$	horizontal compression of the graph of the function $f(x)$ by a factor of $\frac{1}{c}$
$g(x)=b^{cx}$, for $0<c<1$	horizontal stretch of the graph of the function $f(x)$ by a factor of $\frac{1}{c}$

Vertical and Horizontal Reflections of Exponential Functions

Consider the parent exponential function $f(x) = b^x$, where b is a positive real number not equal to 1.

Function	Vertical and Horizontal Reflections
$g(x) = -b^x$	reflection of the graph of the function $f(x)$ over the x-axis
$g(x) = b^{-x}$	reflection of the graph of the function $f(x)$ over the y-axis

Logarithmic Notation Lesson

Logarithmic Functions

For a base b, where $b > 0$, and $b \neq 1$, $y = \log_b x$ if and only if $x = b^y$.

For $f(x) = \log_b x$, the domain is $(0, \infty)$ and the range is $(-\infty, \infty)$.

Properties of Logarithms

- $\log_b 1 = 0$
- $\log_b b = 1$
- $\log_b b^x = x$
- $b^{\log_b x} = x$

Properties of Natural Logarithms

- ln 1 = 0
- ln *e* = 1
- $\ln e^x = x$
- $e^{\ln x} = x$

Graphs of Logarithmic Functions Lesson

Properties of Exponential and Logarithmic Functions, $b > 0$, $b \neq 1$

	Exponential Function	**Logarithmic Function**
function form	$f(x) = b^x$	$f(x) = \log_b x$
domain	$(-\infty, \infty)$	$(0, \infty)$
range	$(0, \infty)$	$(-\infty, \infty)$
asymptote	x-axis	y-axis
x-intercept	none	$(1, 0)$
y-intercept	$(0, 1)$	none

Vertical and Horizontal Translations of Logarithmic Functions

Consider the parent logarithmic function $f(x) = \log_b x$, where b is a positive real number not equal to 1.

Function	**Vertical and Horizontal Translation**
$g(x) = \log_b(x+h)$ for $h > 0$	translation of the graph of the function $f(x)$ h units to the left
$g(x) = \log_b(x-h)$ for $h > 0$	translation of the graph of the function $f(x)$ h units to the right
$g(x) = \log_b(x) + k$ for $k > 0$	translation of the graph of the function $f(x)$ k units up
$g(x) = \log_b(x) - k$ for $k > 0$	translation of the graph of the function $f(x)$ k units down

Vertical and Horizontal Stretches and Compressions of Logarithmic Functions

Consider the parent logarithmic function $f(x)=\log_b x$, where b is a positive real number not equal to 1.

Function	Vertical and Horizontal Stretch and Compression
$g(x)=a\log_b x$ for $a>0$	vertical stretch of the graph of the function $f(x)$ by a factor of a
$g(x)=a\log_b x$ for $0<a<1$	vertical compression of the graph of the function $f(x)$ by a factor of a
$g(x)=\log_b(cx)$ for $c>1$	horizontal compression of the graph of the function $f(x)$ by a factor of $\frac{1}{c}$
$g(x)=\log_b(cx)$ for $0<c<1$	horizontal stretch of the graph of the function $f(x)$ by a factor of $\frac{1}{c}$

Reflections of Logarithmic Functions

Consider the parent logarithmic function $f(x)=\log_b x$, where b is a positive real number not equal to 1.

Function	Vertical and Horizontal Reflections
$g(x)=-\log_b x$	reflection of the graph of the function $f(x)$ over the x-axis
$g(x)=\log_b(-x)$	reflection of the graph of the function $f(x)$ over the y-axis

Comparing Logarithmic Functions

Two logarithmic functions, $f(x)=\log_a x$ and $g(x)=\log_b x$ where $a>0$ and $b>0$ may be compared in the following manner.

If both bases are greater than 1, or both bases are between 0 and 1 and $a>b$, the following applies:

- The graph of *f* lies above the graph of *g* over the interval $(0,1)$.
- The graphs intersect at $x=1$.
- The graph of *f* lies below the graph of *g* when $x>1$.

If base a is greater than 1 but base b is less than 1, the following applies:

- The graph of f lies below the graph of g over the interval $(0,1)$.
- The graphs intersect at $x = 1$.
- The graph of f lies above the graph of g when $x > 1$.

Logarithmic Rules and Solving Logarithmic Equations Lesson

Logarithm Rules

product rule: $\log_b(AB) = \log_b A + \log_b B$

quotient rule: $\log_b\left(\frac{A}{B}\right) = \log_b A - \log_b B$

power rule: $\log_b A^r = r\log_b A$

corollary to the power rule: $\log_b\left(\frac{1}{A}\right) = -\log_b(A)$

Solving Logarithmic Equations Using the Definition of Logarithms

1. Use the rules of logarithms to rewrite the equation in the form $\log_b M = c$.
2. Convert the equation to exponential form: $\log_b M = c \rightarrow b^c = M$.
3. Solve for the variable.
4. Check the solution to ensure only values for which $M > 0$ are included.

Solving Logarithmic Equations Using the One-to-One Property

1. Write the equation in the form $\log_b M = \log_b N$.
2. Set the arguments equal to one another: $M = N$.
3. Solve for the variable.
4. Check the solution to ensure only values for which $M > 0$ and $N > 0$ are included.

Solving Exponential Equations Lesson

Solving Exponential Equations Using a Common Base

1. Rewrite the equation so that both sides have the same base: $b^M = b^N$.
2. Set the exponents equal to one another: $M = N$.
3. Solve for the variable.

Solving Exponential Equations Using Logarithms

If both sides of the equation cannot be written with a common base, use the following steps:

1. Isolate the exponential expression.
2. Take the log of each side of the equation.
3. Simplify using the rules for logarithms.
4. Solve for the variable.

Change of Base Formula

$$\log_b A = \frac{\log A}{\log b} = \frac{\ln A}{\ln b}$$

Piecewise Functions Lesson

Evaluating a Piecewise Function

Use the following steps to evaluate a piecewise function at $x = c$, where c is a real number:

- Locate the interval where c occurs in the partition of the domain.
- Evaluate $f(c)$ using the equation of the function corresponding to the interval where c is located.

Finding the Equation of a Piecewise Function Using the Function's Graph

1. Locate the values of the domain where the function begins, ends, or changes from one equation to another. These values are used to determine the endpoints of the domains for each equation that comprises the piecewise function.
2. Notice that the points on the graph that correspond to the values in step 1 are marked with either an open or closed circle. Open circles on the graph correspond to either a $<$ or $>$ sign in the function. Closed circles on the graph correspond to either a $\leq$, $\geq$ or = sign in the function.
3. Once the domain of the function has been separated into different pieces, study the graph of the equation for each domain to determine if it is a constant, linear, quadratic, or other equation. Write the equation for each piece of the domain.