Conic Sections Key Concepts

Analytic Geometry Lesson

Distance Formula

The distance, *d*, between two points on a coordinate plane with coordinates (x_1, y_1) and (x_2, y_2) is given by the formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

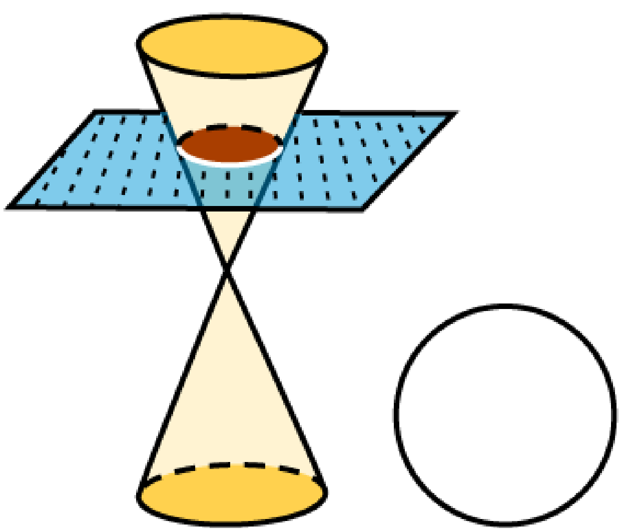
Midpoint of a Line Segment

The midpoint of $\overline{P_1P_2}$, where the coordinates of P_1 are (x_1, y_1) and the coordinates of P_2 are (x_2, y_2), has coordinates $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

Introduction to Conic Sections Lesson

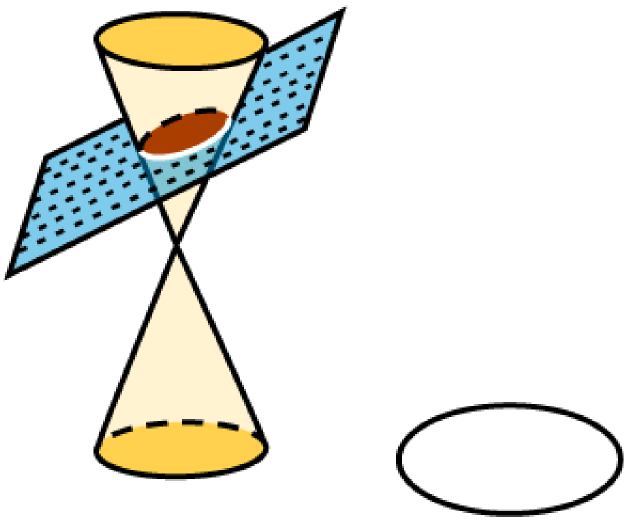
Circle

A **circle** is a conic section formed by slicing a right circular cone with a plane that is perpendicular to its axis.



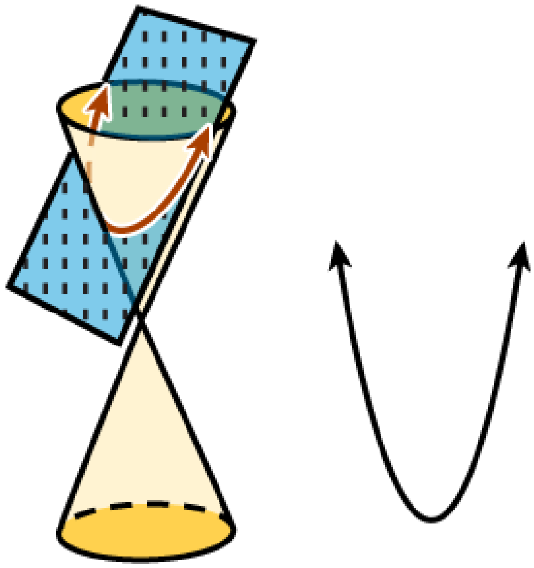
Ellipse

An **ellipse** is a conic section formed by slicing a right circular cone with a plane that is not parallel to its edge nor perpendicular or parallel to its axis.



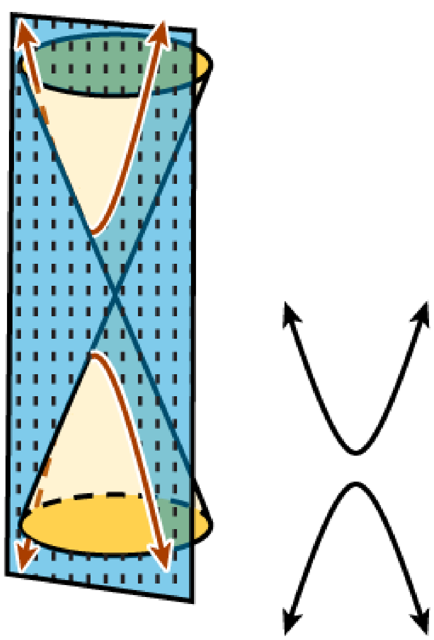
Parabola

A **parabola** is a conic section formed by slicing a right circular cone with a plane that is parallel to its edge.



Hyperbola

A **hyperbola** is a conic section formed by slicing a double cone with a plane that is parallel to the axis of the cones and does not pass through the apexes.



General Equation of a Conic Section

The equation of every conic can be written in the following form:

$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where *A*, *B*, *C*, *D*, *E*, and *F* are real numbers.

Identifying a Conic Section from Its General Equation

The discriminant of the equation is $B^2 - 4AC$.

- If $B^2 - 4AC > 0$, the conic is a hyperbola.
- If $B^2 - 4AC = 0$, the conic is a parabola.
- If $B^2 - 4AC < 0$ and *A* = *C*, the conic is a circle.
- If $B^2 - 4AC < 0$ and $A \neq C$, the conic is an ellipse.

Circles Lesson

Standard Form of a Circle

The standard form of the equation of a circle is $(x-h)^2 + (y-k)^2 = r^2$, where the center is (h,k), any point on the circle is (x,y), and the radius is *r*.

General Form of a Circle

The general form of the equation of a circle is $Ax^2 + Cy^2 + Dx + Ey + F = 0$, where *A*, *C*, *D*, *E*, and *F* are real numbers, *A* = *C*, $A \neq 0$ and $C \neq 0$.

Ellipses Lesson

Standard Form of an Ellipse

- The standard form of the equation of a horizontal ellipse is $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$, with center (h,k), a major axis parallel to the *x*-axis, vertices $(h \pm a, k)$, and foci $(h \pm c, k)$.

- The standard form of the equation of a vertical ellipse is $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$ with center (h,k), a major axis parallel to the *y*-axis, vertices $(h, k \pm a)$, and foci $(h, k \pm c)$.

- For both equations, the length of the major axis is 2*a* and the length of the minor axis is 2*b*, where $a > b > 0$.
- For both equations, $c^2 = a^2 - b^2$. For both equations, the vertices lie on the major axis, *a* units from the center, and the foci lie on the major axis, *c* units from the center.

General Form of an Ellipse

The general form of the equation of an ellipse is $Ax^2 + Cy^2 + Dx + Ey + F = 0$, with $A > 0$, $C > 0$, $A \neq C$, and real numbers *D*, *E*, and *F*.

Parabolas Lesson

Standard Form of a Parabola

- The standard form of the equation of a parabola with a horizontal directrix and a vertical axis of symmetry is $(x-h)^2 = 4p(y-k);\ p \neq 0$ with vertex (h,k), directrix $y = k - p$, and focus $(h, k+p)$.
- The standard form of the equation of a parabola with a vertical directrix and a horizontal axis of symmetry is $(y-k)^2 = 4p(x-h);\ p \neq 0$ with vertex (h,k), directrix $x = h - p$, and focus $(h+p, k)$.

General Form of a Parabola

- The general form of the equation of a parabola with a horizontal directrix is $Ax^2 + Dx + Ey + F = 0$, where *A*, *D*, *E,* and *F* are real numbers.
- The general form of the equation of a parabola with a vertical directrix is $Cy^2 + Dx + Ey + F = 0$, where *C*, *D*, *E,* and *F* are real numbers.

Hyperbolas Lesson

Standard Form of a Hyperbola

- The standard form of the equation of a hyperbola with a horizontal transverse axis is $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$, with center (h,k). The vertices

$(h \pm a, k)$ and foci $(h \pm c, k)$ lie on the axis of symmetry. The equations of the asymptotes are $y = \frac{b}{a}(x-h)+k$, and $y = -\frac{b}{a}(x-h)+k$.

- The standard form of the equation of a hyperbola with a vertical transverse axis is $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$, with center (h,k). The vertices $(h, k \pm a)$ and foci $(h, k \pm c)$ lie on the axis of symmetry. The equations of the asymptotes are $y = \frac{a}{b}(x-h)+k$, and $y = -\frac{a}{b}(x-h)+k$.
- The transverse axis has endpoints at the vertices, which lie *a* units from the center, and the conjugate axis has endpoints *b* units from the center. The foci lie on the axis of symmetry, *c* units from the center.
- For both equations, $c^2 = a^2 + b^2$.

General Form of a Hyperbola

- The general form of the equation of a hyperbola with a horizontal transverse axis is $Ax^2 - Cy^2 + Dx + Ey + F = 0$, with $A > 0$, $C > 0$, $A \neq C$, and real numbers *A*, *C*, *D*, *E* and *F*.
- The general form of the equation of a hyperbola with a vertical transverse axis is $-Ax^2 + Cy^2 + Dx + Ey + F = 0$, with $A > 0$, $C > 0$, $A \neq C$, and real numbers *A*, *C*, *D*, *E* and *F*.