Sequences and Series Key Concepts

Sequences Lesson

Sequences

A sequence, denoted $\{a_n\}$, can be defined using an explicit formula or a recursive formula. An explicit formula provides an equation for the *n*th term of the sequence, denoted a_n. A recursive formula provides a term, or terms, of the sequence, usually a_1, and an equation to find each successive term.

Arithmetic Sequences Lesson

Recursive Formula for an Arithmetic Sequence

An arithmetic sequence can be defined recursively by the formula $a_n = a_{n-1} + d$, where *d* is the common difference, *n* is the set of positive integers, and a_1 is the first term of the sequence.

Explicit Formula for an Arithmetic Sequence

An arithmetic sequence can be defined explicitly by the formula $a_n = a_1 + (n-1)d$, where *d* is the common difference, *n* is the set of positive integers, and a_1 is the first term of the sequence.

Terms of an Arithmetic Sequence Lesson

Arithmetic Mean Formula

$$\overline{x} = \frac{x_1 + x_2 + \ldots + x_n}{n}$$

Finite Arithmetic Series Lesson

Sum of the First *n* Terms of an Arithmetic Sequence

The sum of the first *n* terms of an arithmetic sequence is written S_n, where the following applies:

- S_n is called the *n*th partial sum.
- $S_n = a_1 + a_2 + a_3 + \ldots + a_n$

Finite Arithmetic Sum Formula

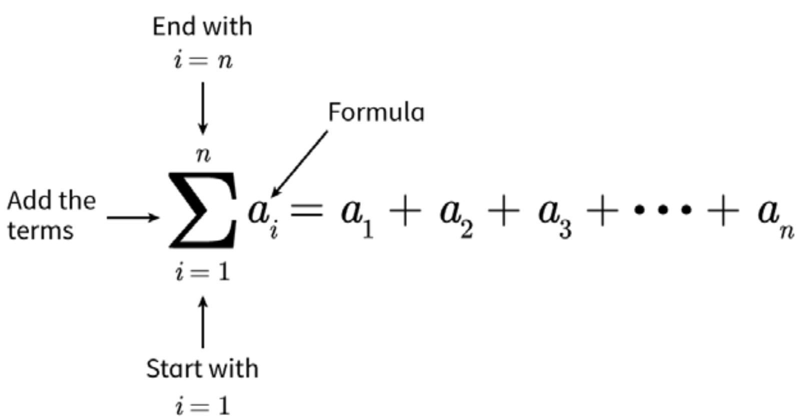
The formula for the sum of the first *n* terms of an arithmetic sequence is

$S_n = \frac{n}{2}(a_1 + a_n)$, where the following applies:

- *n* is the number of terms to be added.
- a_1 is the first term in the sequence.
- a_n is the *n*th term in the sequence.

Arithmetic Series Summation Formulas Lesson

Sigma Notation



If *N* represents the number of terms added by the sum $\sum_{i=x}^{n} a_i$, then $N = n - x + 1$.

Properties of Summation

1. If *c* is a real number, $\sum_{i=1}^{n} c = nc$.

2. If c is a real number, $\sum_{i=1}^{n} cx_i = c\sum_{i=1}^{n} x_i$.

3. $\sum_{i=1}^{n}\left(x_i + y_i\right) = \sum_{i=1}^{n} x_i + \sum_{i=1}^{n} y_i$

4. $\sum_{i=1}^{n}\left(x_i - y_i\right) = \sum_{i=1}^{n} x_i - \sum_{i=1}^{n} y_i$

Summation Formulas

1. $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

2. $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

3. $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

Equations of a Geometric Sequence Lesson

Recursive Formula for a Geometric Sequence

A geometric sequence can be defined recursively by the formula $a_n = a_{n-1} \cdot r$, where *r* is the common ratio, *n* is the set of positive integers, and a_1 is the first term of the sequence.

Explicit Formula for a Geometric Sequence

A geometric sequence can be defined explicitly by the formula $a_n = a_1 r^{n-1}$ where *r* is the common ratio, *n* is the set of positive integers, and a_1 is the first term of the sequence.

Terms of a Geometric Sequence Lesson

Geometric Mean Formula

$GM = \sqrt[n]{x_1 \cdot x_2 \cdot \ldots \cdot x_n}$

Convergent and Divergent Sequences and Series Lesson

Convergence Test for Geometric Series

The geometric series $a_n = a_1 r^{n-1}$ converges if $|r| < 1$ and diverges if $|r| \geq 1$.

Finite Geometric Series Lesson

Sum of the First *n* Terms of a Geometric Sequence

The sum of the first *n* terms of a geometric sequence is written S_n, where the following applies:

- S_n is called the *n*th partial sum.
- $S_n = a_1 + a_2 + a_3 + \ldots + a_n$

Finite Geometric Sum Formula

The formula for the sum of the first *n* terms of a geometric sequence $\{a_n\}$ is $S_n = \frac{a_1(1-r^n)}{1-r}, r \neq 1$, where the following applies:

- *n* is the number of terms to be added.
- a_1 is the first term in the sequence.
- *r* is the common ratio.

Infinite Geometric Series Lesson

Sum of an Infinite Geometric Series

The sum of an infinite geometric series is given by S_∞, where $S_\infty = a_1 + a_2 + a_3 + \ldots$, which can also be expressed as $S_\infty = \sum_{i=1}^{\infty}\left(a_1 \cdot r^{i-1}\right)$.

Infinite Geometric Sum Formula

The formula for the sum of an infinite geometric series is $S_\infty = \frac{a_1}{1-r}$, where the following applies:

- a_1 is the first term.
- *r* is the common ratio.
- $|r| < 1$