UNIT 1 LESSONS 1 – 3

PRECALCULUS B

“TRIGONOMETRY”

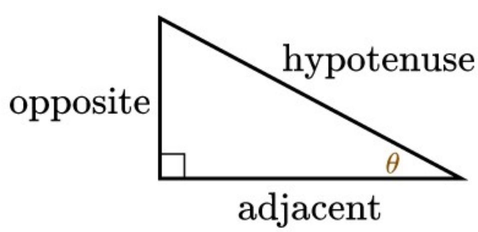
means

“TRIANGLE MEASURE”

LESSONS:

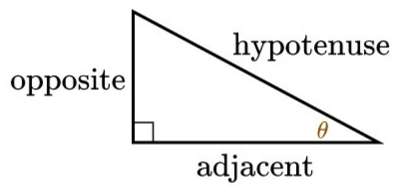
- Right Triangle Trigonometry
- Solving Right Triangles
- Angle Measurements

RIGHT TRIANGLE:



. . . That angle symbol is the Greek letter "theta".

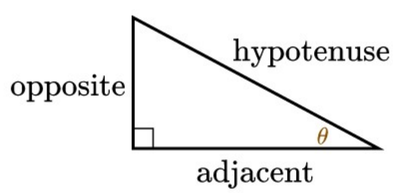
TRIG RATIOS: SOH-CAH-TOA



SOH = ?
CAH = ?
TOA = ?

HINT:
Ratio means a Fraction!

TRIG RATIOS: SOH-CAH-TOA

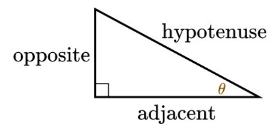


SOH = SINE = opposite/hypotenuse

CAH = COSINE = adjacent/hypotenuse

TOA = TANGENT = opposite/adjacent

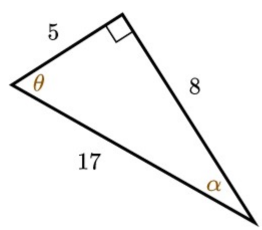
RECIPROCAL Trig Ratios:



- $\text{sine}\,\theta = \sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$
- $\text{cosine}\,\theta = \cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$
- $\text{tangent}\,\theta = \tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$

- $\text{cosecant}\,\theta = \csc\theta = \dfrac{\text{hypotenuse}}{\text{opposite}}$
- $\text{secant}\,\theta = \sec\theta = \dfrac{\text{hypotenuse}}{\text{adjacent}}$
- $\text{cotangent}\,\theta = \cot\theta = \dfrac{\text{adjacent}}{\text{opposite}}$

Practice:



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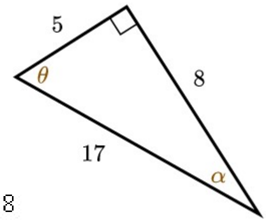
- $\text{cotangent}\,\theta = \cot\theta = \dfrac{\text{adjacent}}{\text{opposite}}$

. . . **BTW, that other angle symbol is the Greek letter "alpha".**

Practice:



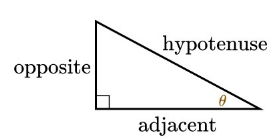
- $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{8}{17}$
- $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}} = \dfrac{5}{17}$
- $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{8}{5}$
- $\csc\theta = \dfrac{\text{hypotenuse}}{\text{opposite}} = \dfrac{17}{8}$
- $\sec\theta = \dfrac{\text{hypotenuse}}{\text{adjacent}} = \dfrac{17}{5}$
- $\cot\theta = \dfrac{\text{adjacent}}{\text{opposite}} = \dfrac{5}{8}$

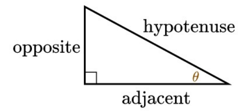
Trig COFUNCTIONS:

. . . When functions of complementary angles are equal to each other.

Quick!

What are complementary angles???





Trig COFUNCTIONS:

. . . When functions of complementary angles are equal to each other.

Quick!

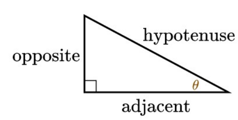
What are complementary angles???

Right!!

Two angles that add up to 90 degrees.

Trig COFUNCTIONS:

. . . When functions of complementary angles are equal to each other.



So . . .

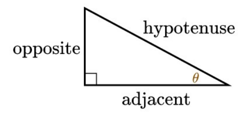
Sine and Cosine are Cofunctions

Tangent and Cotangent are Cofunctions

Secant and Cosecant are Cofunctions

Trig COFUNCTIONS:

. . . When functions of complementary angles are equal to each other.



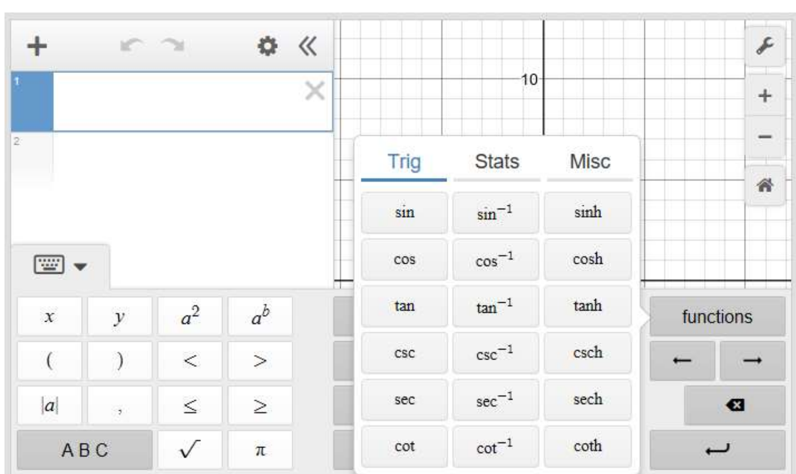
For example:

sin(30) = cos(60)

tan(45) = cot(45)

sec(20) = csc(70)

TRIG Functions on Desmos:



TRIG Functions on other calculators:

. . . When you only have buttons for

sin, cos, tan

. . . But you need to find

cot, sec, csc

Do the Calculation:

1 divided by its reciprocal function!!

For example:

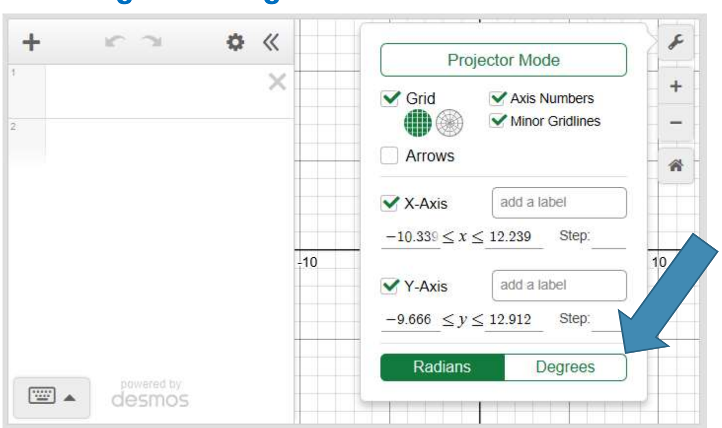
To find csc(30) . . . Do 1 divided by sin(30).

Try it! What do you get?

Check if you are using Degrees or Radians,

And make sure your calculator matches!!!!

. . . Desmos defaults to Radians, here's where to change it to Degrees!!



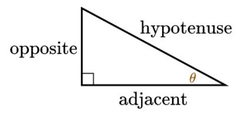
Other calculators usually default to Degrees.

Look for the button "DRG" to change the setting.

Trig COFUNCTIONS:

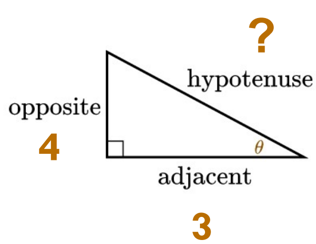
. . . When functions of complementary angles are equal to each other.

For example:



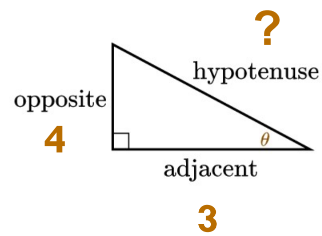
So, how do we use these?

If you know the lengths of two sides, use the Pythagorean Theorem to calculate the third side, then you can get all six trig ratios!!



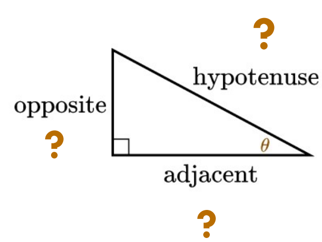
If you know one trig ratio,
That gives you two of the sides!!

Instead of this diagram, you could get the same information by being told that tan θ = 4/3.



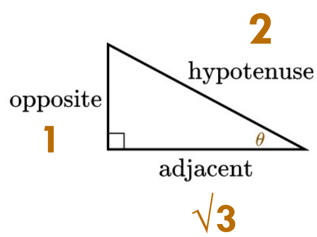
Practice:

If sin θ = 1/2
What are the lengths of the sides?



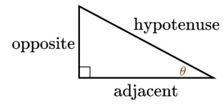
The Pythagorean Theorem will give you the third side!

If $\sin \theta = 1/2$
What are the lengths of the sides?



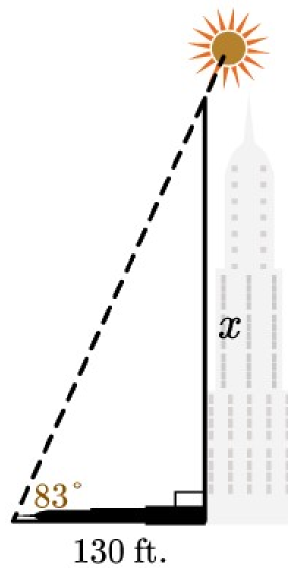
Or, if you know the measure of the angle, and the length of one side, you can calculate the other sides!

The classic example is finding the height of something tall by measuring it's shadow and knowing the angle measure from the tip of the shadow to the top of the of where the shadow came from . . .



Since we want the height, that would be the side opposite of the angle on the ground.

Since we know the side adjacent to that angle, we would use which trig ratio??

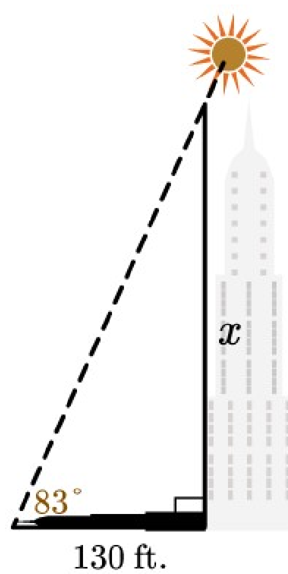


Tangent is opposite over adjacent!

$$\tan 83° = \frac{x}{130}$$

Next, solve for x . . .

Remember to check if your calculator is set to degrees or radians!!!

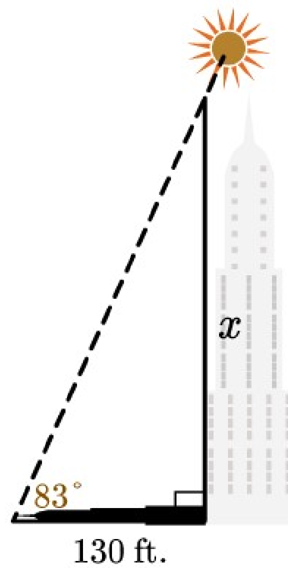


Tangent is opposite over adjacent!

$$\tan 83° = \frac{x}{130}$$

$$8.1443 = \frac{x}{130}$$

$$1{,}058.7650 = x$$



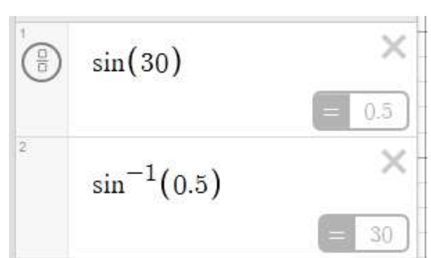
Remember to check if your calculator is set to degrees or radians!!!

INVERSE Trig Functions:

For going in reverse to find the angle that was used to get the ratio!

So when sin θ = x,
The inverse would be arcsin x = θ

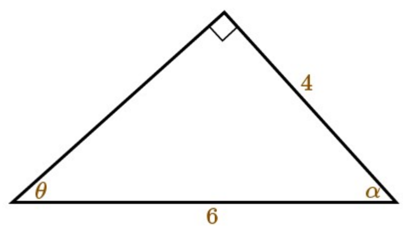
For example:
If sin (30) = 1/2,
Then arcsin (1/2) = 30



Remember to check if your calculator is set to degrees or radians!!!

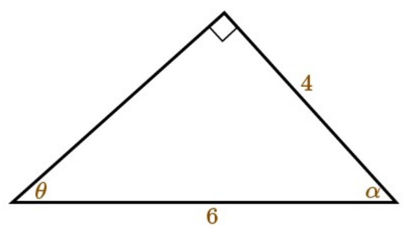
Use the Inverse Trig Functions when you know at least two of the sides, and you want to know the angle!

Practice: Find all the angles



Remember to check if your calculator is set to degrees or radians!!!

Practice: Find all the angles, use the inverse function to solve for θ

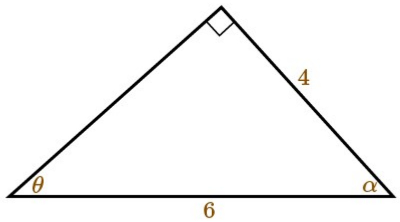


$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin\theta = \frac{4}{6}$$

Remember to check if your calculator is set to degrees or radians!!!

Practice: Find all the angles, use the inverse function to solve for θ



$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin\theta = \frac{4}{6}$$

$$\sin\theta = 0.6667$$

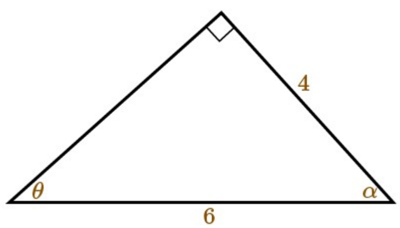
$$\theta = \sin^{-1}(0.6667)$$

$$\theta \approx 41.8103°$$

Next, solve for the measure of angle alpha!

Remember to check if your calculator is set to degrees or radians!!!

Practice: Find all the angles, use the inverse function to solve for θ



$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin\theta = \frac{4}{6}$$

$$\sin\theta = 0.6667$$

$$\theta = \sin^{-1}(0.6667)$$

$$\theta \approx 41.8103°$$

$$\alpha + \theta + 90° = 180°$$

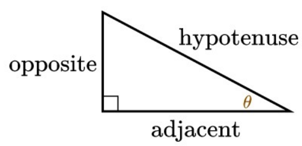
$$\alpha + 41.8103° + 90° = 180°$$

$$\alpha + 131.8103° = 180°$$

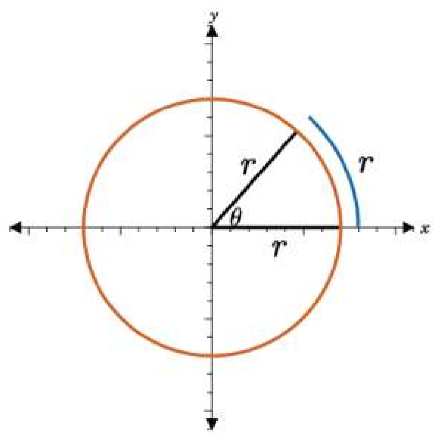
$$\alpha \approx 48.1897°$$

Solving a Right Triangle for all sides and angles:

- ✓ If you are given information that tells you two of the sides,
- ✓ Use Pythagorean Theorem to solve for the third side,
- ✓ Use Inverse Trig Functions to solve for the angles.

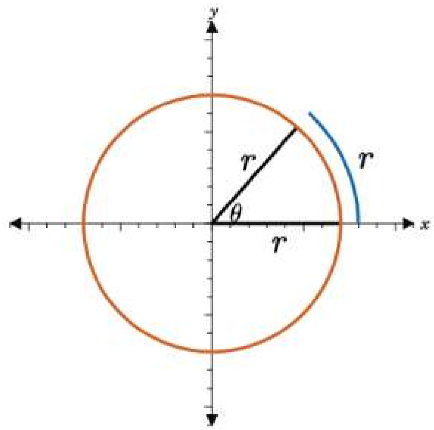


Now, about those Radians . . .



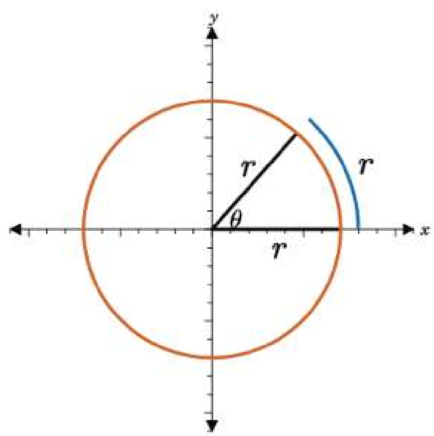
Do you remember what these are???

Radians are a measurement unit for angles.



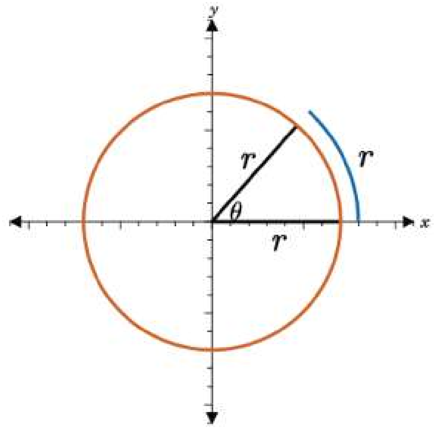
An angle of 1 radian cuts off an arc of length 1 radius.

The circumference of a circle is 2πr.



So, for a radius of 1, the distance around is 2π radians.

2π radians = 360° … or … π radians = 180°



This is your conversion ratio to switch units!!

Practice:

45° = ? Radians

300° = ? Radians

$7\pi/6$ radians = ? Degrees

3π radians = ? Degrees

Remember, π rad = 180°

Practice:

45° = ? Radians

$$45° \cdot \frac{\pi}{180°}$$

$$\frac{45\pi}{180}$$

$$\frac{\pi}{4}$$

300° = ? Radians

$$300° \cdot \frac{\pi}{180°}$$

$$\frac{300\pi}{180}$$

$$\frac{5\pi}{3}$$

Remember, π rad = 180°

Practice:

7π/6 rad = ? Deg

$$\frac{7\pi}{6} \cdot \frac{180°}{\pi}$$

$$\frac{1,260\pi}{6\pi}$$

$$210°$$

3π rad = ? Deg

$$3\pi \cdot \frac{180°}{\pi}$$

$$\frac{540\pi}{\pi}$$

$$540°$$

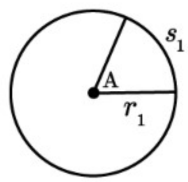
Remember, π rad = 180°

One more way to get the measurement of an angle:

. . . But this only works with radians!

An angle (in radians) = $\frac{\text{arc length}}{\text{radius}}$

$$\theta = s/r$$



Practice with $\theta = s/r$

Given an arc length of 15 and a radius of 6,
How many radians is the central angle?
How many degrees?

Given a central angle of 45° and a radius of 8,
How long is the arc?

Given an arc length of 15 and a radius of 6,
How many radians is the central angle?

$$\theta = \frac{s}{r}$$

$$\theta = \frac{15}{6}$$

$$\theta = 2.5 \text{ radians}$$

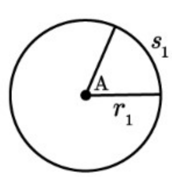
How many degrees?

$$\theta = 2.5 \cdot \frac{180}{\pi} = \frac{450}{\pi} \approx 143.2396°$$

Given a central angle of 45° and a radius of 8,
How long is the arc?

TIPS:

- **You must convert to radians to use this formula.**
- **Any time you know all but one number in a formula, you can use your algebra skills to solve for the one unknown.**



Given a central angle of 45° and a radius of 8, How long is the arc?

$$45^\circ \cdot \frac{\pi}{180^\circ} = \frac{\pi}{4}$$

$$\theta = \frac{s}{r}$$

$$\frac{\pi}{4} = \frac{s}{8}$$

$$s = 6.3 \text{ m}$$

First, change degrees to radians so you can use the formula!

In Summary:

Trig Ratios can be paired as Cofunctions or as Reciprocal Functions.

With two sides of a right triangle, you can calculate the third using the Pythagorean Theorem, and then find all six trig ratios.

With one side and one angle of a right triangle, you can calculate a second side (like the shadow problem), and then find the rest.

The Inverse function solves for the angle.

With two sides of a right triangle, you can use an inverse function to calculate an angle. Then, since all three angles add to 180, you can find the other angles.

Convert between radians and degrees using π radians = 180 degrees.

The measure of a central angle = arc length/radius.

Questions??

Review the Key Terms and Key Concepts documents for this unit.

Look up the topic at khanacademy.org and virtualnerd.com

Check our class website at nca-patterson.weebly.com

*Reserve a time for a call with me at
jpattersonmath.youcanbook.me

We can use the LiveLesson whiteboard to go over problems together!