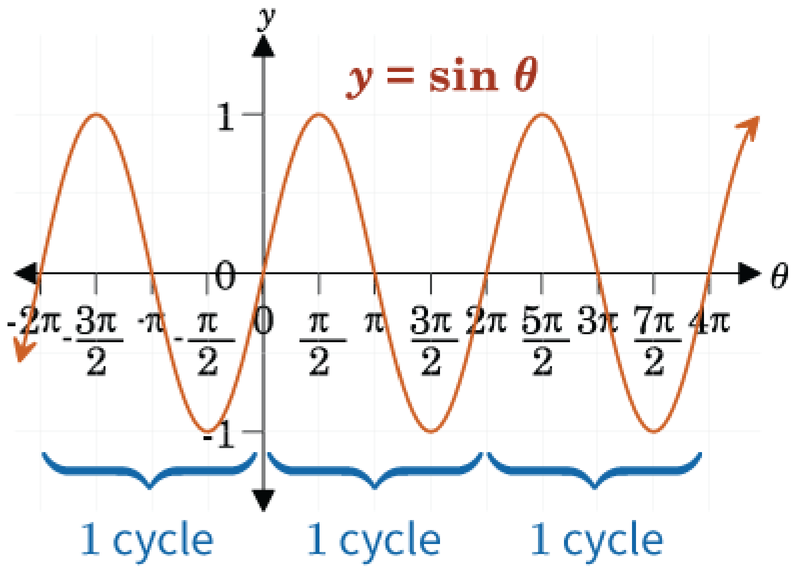
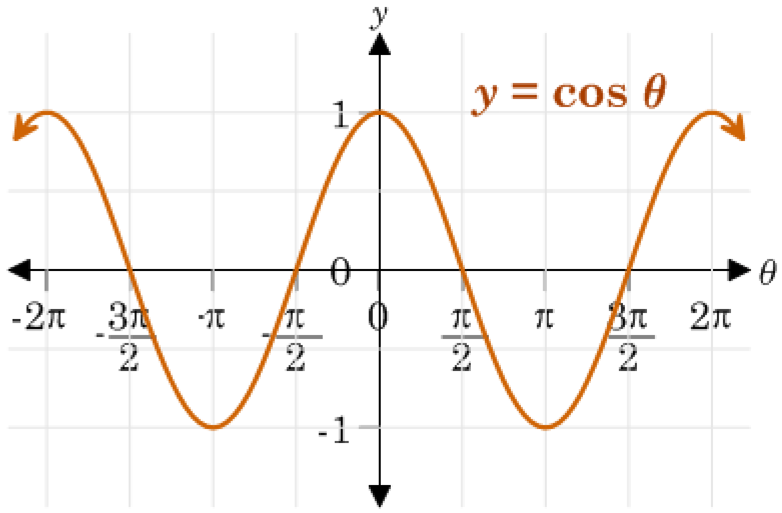
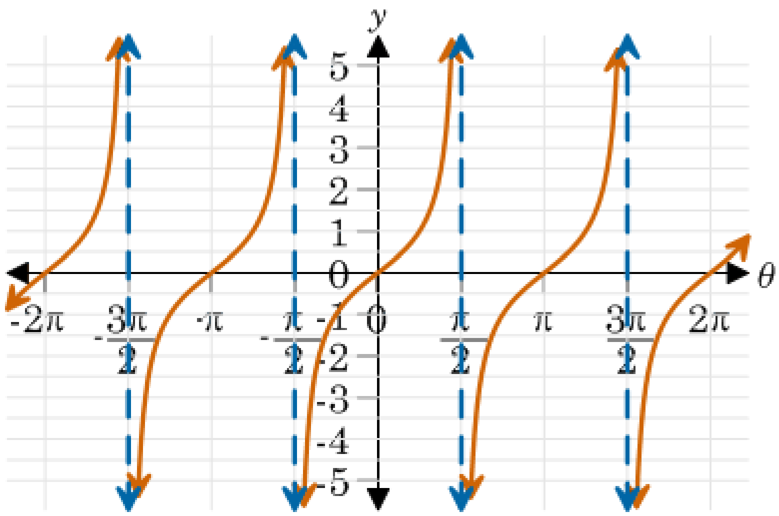
Trigonometric Functions Key Concepts

Graphs of Trigonometric Functions Lesson

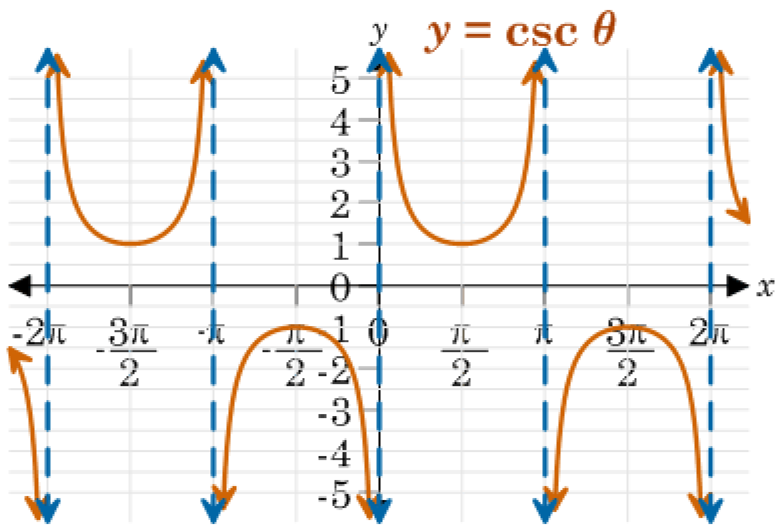
Graphs of Sine, Cosine, and Tangent

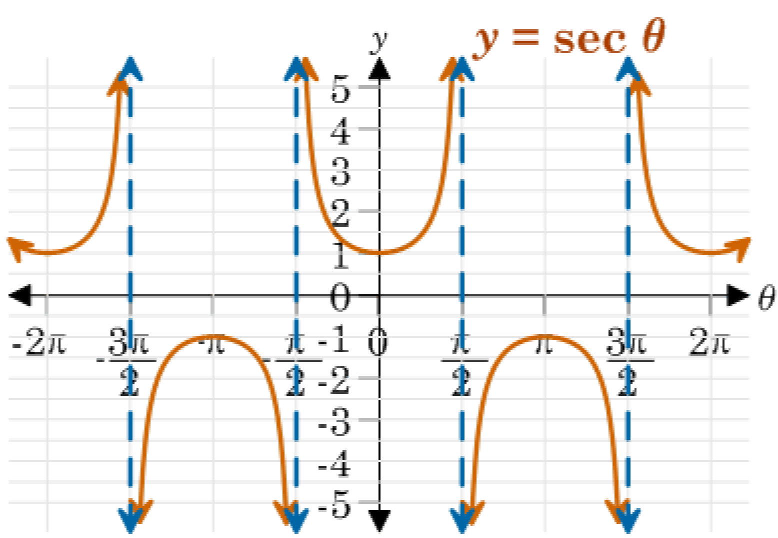


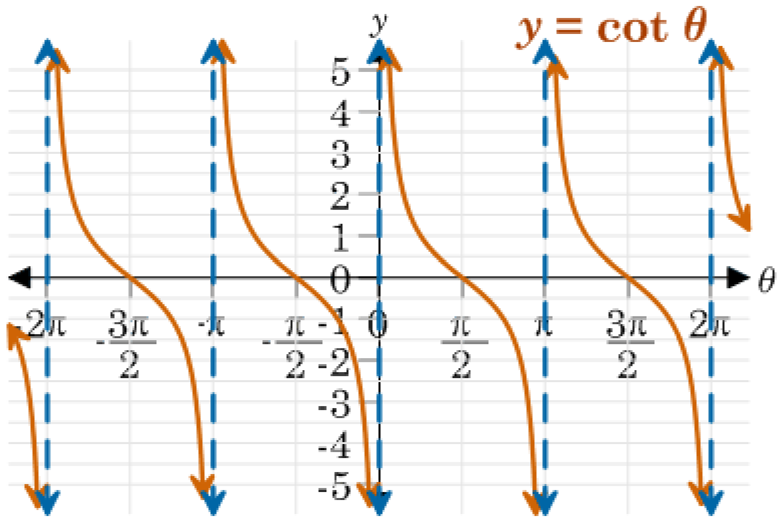




Graphs of Cosecant, Secant, and Cotangent







Domain and Range of Trigonometric Functions Lesson

Domain and Range of Trigonometric Functions

The following are the domain and range of the six basic trigonometric functions:

Function	Domain	Range
y = sin x	$(-\infty,\infty)$	$[-1,1]$
y = cos x	$(-\infty,\infty)$	$[-1,1]$
y = tan x	$(-\infty,\infty)$, except $\frac{\pi}{2}+n\pi$, where n is an integer	$(-\infty,\infty)$
y = cot x	$(-\infty,\infty)$, except $n\pi$, where n is an integer	$(-\infty,\infty)$
y = csc x	$(-\infty,\infty)$, except $n\pi$, where n is an integer	$(-\infty,-1]\cup[1,\infty)$
y = sec x	$(-\infty,\infty)$, except $\frac{\pi}{2}+n\pi$, where n is an integer	$(-\infty,-1]\cup[1,\infty)$

Behavior of Trigonometric Functions Lesson

Periodic and Even and Odd Identities for Sine and Cosine

Periodic Identities

- $\sin(\theta + 2\pi n) = \sin\theta$
- $\cos(\theta + 2\pi n) = \cos\theta$

Even and Odd Identities

- $\sin(-\theta) = -\sin\theta$
- $\cos(-\theta) = \cos\theta$

Periodic and Even and Odd Identities for Tangent and Cotangent

Periodic Identities

- $\tan(\theta + \pi n) = \tan\theta$
- $\cot(\theta + \pi n) = \cot\theta$

Even and Odd Identities

- $\tan(-\theta) = -\tan\theta$
- $\cot(-\theta) = -\cot\theta$

Periodic and Even and Odd Identities for Cosecant and Secant

Periodic Identities

- $\csc(\theta + 2\pi n) = \csc\theta$
- $\sec(\theta + 2\pi n) = \sec\theta$

Even and Odd Identities

- $\csc(-\theta) = -\csc\theta$
- $\sec(-\theta) = \sec\theta$

Behavior of Trigonometric Functions

Function	Period	Amplitude	Zeroes (where k is an integer)	Asymptotes (where k is an integer)	Even or Odd?	Symmetry
$f(x) = \sin x$	2π	1	$k\pi$	n/a	odd	about the origin
$f(x) = \cos x$	2π	1	$\frac{\pi}{2} + k\pi$	n/a	even	about the y-axis
$f(x) = \tan x$	π	n/a	$k\pi$	$x = \frac{\pi}{2} + k\pi$	odd	about the origin
$f(x) = \csc x$	2π	n/a	none	$x = k\pi$	odd	about the origin
$f(x) = \sec x$	2π	n/a	none	$x = \frac{\pi}{2} + k\pi$	even	about the y-axis
$f(x) = \cot x$	π	n/a	$\frac{\pi}{2} + k\pi$	$x = k\pi$	odd	about the origin

Properties of Trigonometric Functions Lesson

Period and Amplitude of Trigonometric Functions

Function	Period	Amplitude
$f(x) = a\sin(bx + c) + d$	$\frac{2\pi}{\lvert b\rvert}$	$\lvert a\rvert$
$f(x) = a\cos(bx + c) + d$	$\frac{2\pi}{\lvert b\rvert}$	$\lvert a\rvert$
$f(x) = a\tan(bx + c) + d$	$\frac{\pi}{\lvert b\rvert}$	n/a
$f(x) = a\csc(bx + c) + d$	$\frac{2\pi}{\lvert b\rvert}$	n/a
$f(x) = a\sec(bx + c) + d$	$\frac{2\pi}{\lvert b\rvert}$	n/a
$f(x) = a\cot(bx + c) + d$	$\frac{\pi}{\lvert b\rvert}$	n/a

Phase Shift and Vertical Shift

For a function in the form $y = a \cdot f(bx + c) + d$, where *f* is one of the six trigonometric functions, the phase shift is determined by the expression $-\frac{c}{b}$, where a positive value shifts the graph to the right and a negative value shifts the graph to the left. The vertical shift is determined by *d*, where a positive *d* value translates the graph up and a negative *d* value translates the graph down.

Graphing Transformations of Trigonometric Functions Lesson

Vertical Stretch or Compression

For a trigonometric function in the form *y* = *a* sin *x* or *y* = *a* cos *x*, $|a|$ = amplitude. The graph of *y* = *a* tan *x* does not have an amplitude.

- $|a| > 1$ stretches the graph vertically by a factor of $|a|$.
- $0 < |a| < 1$ compresses the graph vertically by a factor of $|a|$.
- $a < 0$ reflects the graph across the *x*-axis and also stretches the graph.
- For the graph of tangent functions, the points one-fourth and three-fourths of the way between the asymptotes have *y*-coordinates of $-a$ and *a,* respectively.

Horizontal Stretch or Compression

For trigonometric functions in the form *y* = *a* sin *bx* or *y* = *a* cos *bx*, the amplitude is $|a|$ and the period is $\frac{2\pi}{b}$.

For trigonometric functions in the form *y* = *a* tan *bx*, the period is $\frac{\pi}{b}$.

- $|b| > 1$ compresses the graph horizontally to increase the frequency by a factor of $|b|$.
- $0 < |b| < 1$ stretches the graph horizontally to decrease the frequency by a factor of $|b|$.
- $|b| < 0$ reflects the graph across the *y*-axis.

- For the graph of tangent functions, asymptotes occur at $bx = \pm\frac{\pi}{2}$.

Phase Shift

For a trigonometric function in the form $y = a\sin(bx + c)$, $y = a\cos(bx + c)$, or $y = a\tan(bx + c)$, the phase shift equals $-\frac{c}{b}$.

- When $-\frac{c}{b} > 0$, it shifts the graph to the right $\left|\frac{c}{b}\right|$ units.
- When $-\frac{c}{b} < 0$, it shifts the graph to the left $\left|\frac{c}{b}\right|$ units.
- For the graph of tangent functions, asymptotes occur at $bx + c = \pm\frac{\pi}{2}$.

Vertical Shift

For a trigonometric function in the form $y = a\sin(bx + c) + d$, $y = a\cos(bx + c) + d$, or $y = a\tan(bx + c) + d$, *d* represents the vertical shift.

- When $d > 0$, it shifts the graph up *d* units, making the midline *y* = *d.*
- When $d < 0$, it shifts the graph down *d* units, making the midline *y* = *d.*
- For the graph of tangent functions, the points one-fourth and three-fourths of the way between the asymptotes have *y*-coordinates of $-a + d$ and *a* + *d,* respectively.

Writing Equations of Trigonometric Functions Lesson

Steps for Writing an Equation of a Trigonometric Function

1. Identify the parent function and standard form of the transformed graph.
2. Locate the midline and identify the vertical shift.
3. Identify the amplitude.
4. Identify the period and use the formula for period to calculate the value of *b.*
5. Identify the phase shift and use the formula for phase shift with the value of *b* to find the value of *c.*

6. Substitute the values into the standard form of the equation.

Modeling with Trigonometric Functions Lesson

Steps for Mathematical Modeling

1. Plot the data to identify the pattern.
2. Determine which trigonometric function best models the data.
3. Calculate the amplitude using the maximum and minimum values of the data.
4. Determine the period and calculate *b* using $b = \frac{2\pi}{\text{period}}$.
5. Identify the horizontal shift and calculate *c* using $-\frac{c}{b}$.
6. Determine the vertical shift by averaging the maximum and minimum values.
7. Substitute the values into the standard equation.

Inverse Trigonometric Functions Lesson

Standard Restricted Domains

A function and its inverse are mirror images reflected about the line *y* = *x*. In order for a trigonometric function to be invertible, the domain must be restricted. This table shows the standard domain restrictions for the six main trigonometric functions.

Function	Domain	Range
sin *x*	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[-1,1]$
cos *x*	$[0,\pi]$	$[-1,1]$
tan *x*	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$	$(-\infty,\infty)$
cot *x*	$(0,\pi)$	$(-\infty,\infty)$

sec *x*	$\left[0,\frac{\pi}{2}\right)\cup\left(\frac{\pi}{2},\pi\right]$	$(-\infty,-1]\cup[1,\infty)$
csc *x*	$\left[-\frac{\pi}{2},0\right)\cup\left(0,\frac{\pi}{2}\right]$	$(-\infty,-1]\cup[1,\infty)$

Domain and Range of Inverse Trigonometric Functions

Once the domains of the trigonometric functions are restricted and the functions are inverted, then the domain of the trigonometric function becomes the range of the inverse function, and the range of the trigonometric function becomes the domain of the inverse function.

Inverse Trigonometric Function	**Domain**	**Range**
$\sin^{-1} x = \arcsin x$	$[-1,1]$	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$
$\cos^{-1} x = \arccos x$	$[-1,1]$	$[0,\pi]$
$\tan^{-1} x = \arctan x$	$(-\infty,\infty)$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$
$\cot^{-1} x = \operatorname{arccot} x$	$(-\infty,\infty)$	$(0,\pi)$
$\sec^{-1} x = \operatorname{arcsec} x$	$(-\infty,-1]\cup[1,\infty)$	$\left[0,\frac{\pi}{2}\right)\cup\left(\frac{\pi}{2},\pi\right]$
$\csc^{-1} x = \operatorname{arccsc} x$	$(-\infty,-1]\cup[1,\infty)$	$\left[-\frac{\pi}{2},0\right)\cup\left(0,\frac{\pi}{2}\right]$

Evaluating Inverse Trigonometric Functions

Inverse Trigonometric Function	**Equivalent Form**	**Range**
$\theta = \sin^{-1} x$	sin θ = *x*	$-\frac{\pi}{2}\le\theta\le\frac{\pi}{2}$
$\theta = \cos^{-1} x$	cos θ = *x*	$0\le\theta\le\pi$
$\theta = \tan^{-1} x$	tan θ = *x*	$-\frac{\pi}{2}<\theta<\frac{\pi}{2}$

$\theta = \cot^{-1} x$	cot θ = x	$0 < \theta < \pi$
$\theta = \sec^{-1} x$	sec θ = x	$0 \le \theta \le \pi$ and $\theta \ne \frac{\pi}{2}$
$\theta = \csc^{-1} x$	csc θ = x	$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$ and $\theta \ne 0$

Composition of Trigonometric Functions Lesson

Inverse Properties of Trigonometric Functions

If $-1 \le x \le 1$ and $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$, then $\sin(\arcsin x) = x$ and $\arcsin(\sin y) = y$.

If $-1 \le x \le 1$ and $0 \le y \le \pi$, then $\cos(\arccos x) = x$ and $\arccos(\cos y) = y$.

If x is a real number and $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$, then $\tan(\arctan x) = x$ and $\arctan(\tan y) = y$.