UNIT 2 LESSONS 3-5

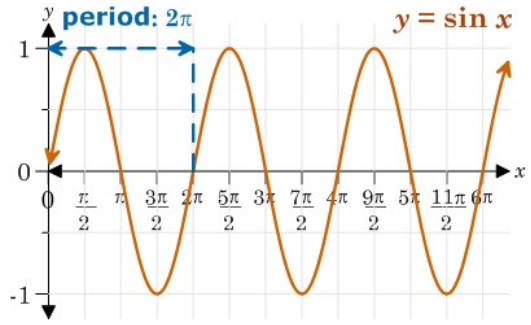
PRECALCULUS B

Behavior & Transformations of Trig Functions:

- Period
- Amplitude
- Frequency
- Phase Shift
- Vertical Shift

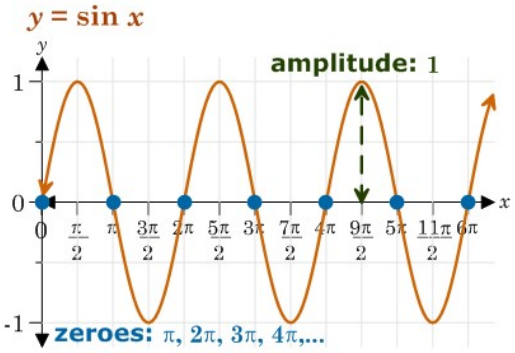
PERIOD

The horizontal length of one cycle of the pattern is the PERIOD of the function.



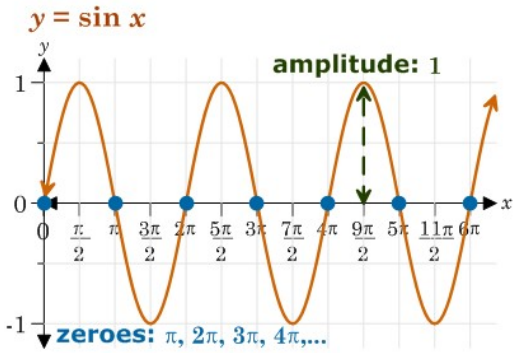
AMPLITUDE

The vertical height from the mid-line to the max or min is the AMPLITUDE.



Remember - OSCILLATE

The function oscillates about the mid-line, alternating between the max and the min.



Period & Amplitude

Both can have a stretch/compress factor.
f(x) = a sin(bx) or f(x) = a cos(bx)

The amplitude is "a".
… a vertical stretch/compress

The number of cycles in one of the standard periods is "b".
… a horizontal stretch/compress

Period & Amplitude

Note that "b" is not the period!

It is the number of cycles in one of the standard periods for that function after the horizontal stretch/compress.

For sin & cos, their standard period is 2π

FREQUENCY

When you change the number of cycles in the standard period distance, you've changed the frequency through a horizontal stretch or compress.

Frequency is the reciprocal of the Period.

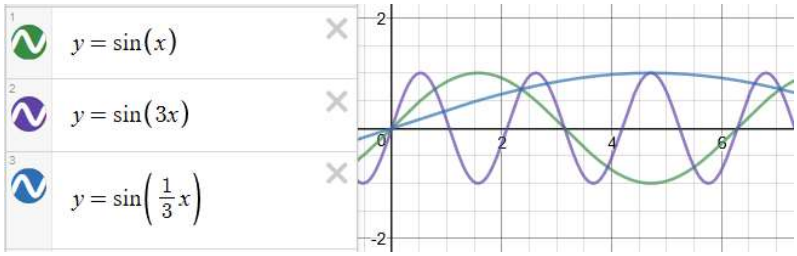
A horizontal compression means more cycles in one interval. So, $|b|>1$.

A horizontal stretch means fewer cycles in one interval. So, $|b|<1$.

FREQUENCY

A horizontal compression increases the frequency. $|b|>1$.

A horizontal stretch decreases the frequency. $|b|<1$.



Period & Amplitude

TRY: 3 cos (2x)

Amplitude = ?

Period = ?

Period & Amplitude

TRY: 3 cos (2x)

Amplitude = 3

Period = 1π

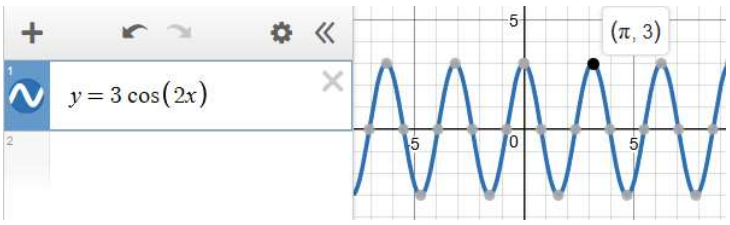
… because 2 cycles in the standard 2π means one cycle is half that length

Period & Amplitude

Graph: 3 cos (2x)

Amplitude = 3 … a vertical stretch

Period = 1π … a horizontal compress



Period & Amplitude

TRY: 5 sin (x/2)

Amplitude = ?
Period = ?

Period & Amplitude

TRY: 5 sin (x/2)

Amplitude = 5
Period = 4π

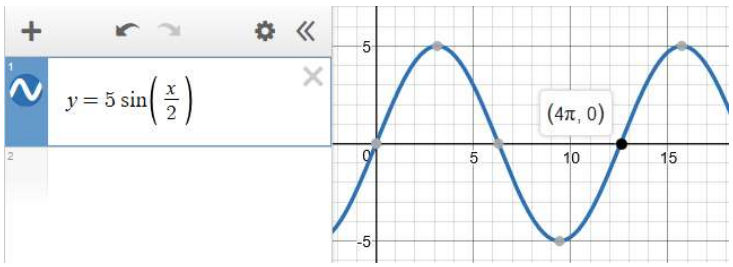
… because 1/2 cycle in the standard 2π means one cycle is double that length

Period & Amplitude

Graph: 5 sin (x/2)

Amplitude = 5 … a vertical stretch

Period = 4π … a horizontal stretch

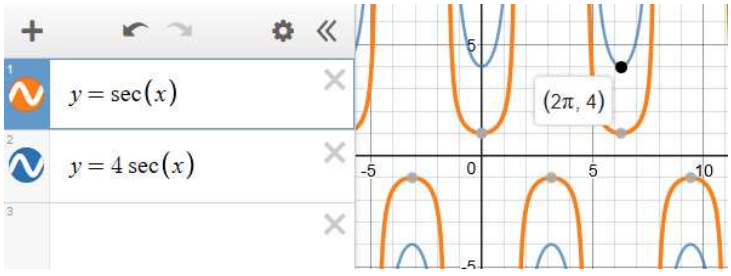


Period & Amplitude

TRY: 4 sec (x)

Amplitude = ?

Period = ?

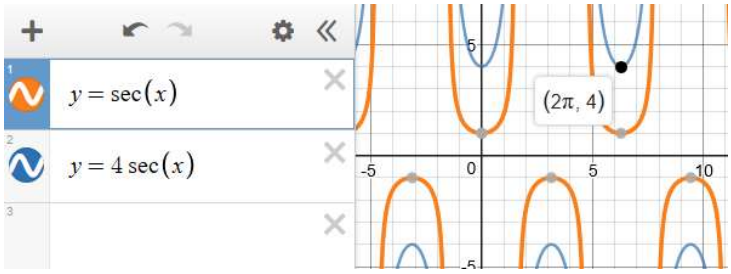


Period & Amplitude

TRY: 4 sec (x)

Amplitude = no max/min, so no amplitude

Period = same as standard = 2π

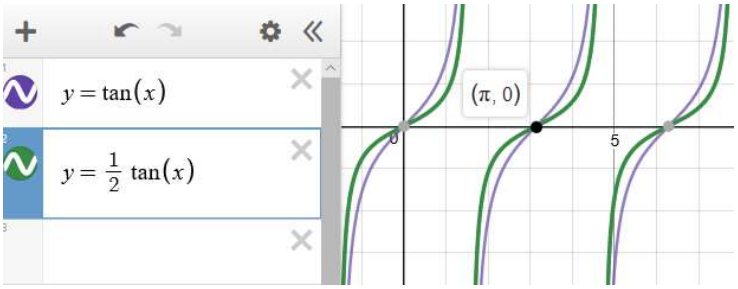


Period & Amplitude

TRY: ½ tan (x)

Amplitude = ?

Period = ?

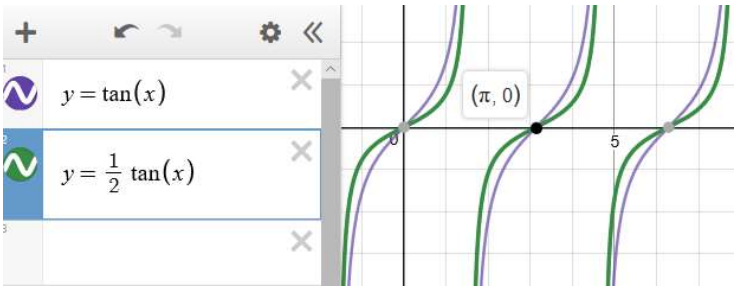


Period & Amplitude

TRY: ½ tan (x)

Amplitude = no max or min = no amplitude

Period = same as standard = 1π



Period & Amplitude

All the trigonometric functions:

Function	Period	Amplitude
$f(x) = \sin x$	2π	1
$f(x) = \cos x$	2π	1
$f(x) = \tan x$	π	n/a
$f(x) = \csc x$	2π	n/a
$f(x) = \sec x$	2π	n/a
$f(x) = \cot x$	π	n/a

Period & Amplitude

All the trigonometric functions:

Function	Period	Amplitude
$f(x) = \sin x$	2π	1
$f(x) = \cos x$	2π	1
$f(x) = \tan x$	π	n/a
$f(x) = \csc x$	2π	n/a
$f(x) = \sec x$	2π	n/a
$f(x) = \cot x$	π	n/a

Periodic Identities
$\sin(\theta + 2\pi n) = \sin\theta$
$\cos(\theta + 2\pi n) = \cos\theta$

Periodic Identities
$\tan(\theta + \pi n) = \tan\theta$
$\cot(\theta + \pi n) = \cot\theta$

Periodic Identities
$\csc(\theta + 2\pi n) = \csc\theta$
$\sec(\theta + 2\pi n) = \sec\theta$

Period & Amplitude

All the trigonometric functions:

So, whatever the answer is for sin θ, will be the same answer for θ+2π, and for θ+4π, and so forth, because it's just another cycle around the unit circle!

Periodic Identities
$\sin(\theta + 2\pi n) = \sin\theta$
$\cos(\theta + 2\pi n) = \cos\theta$

Periodic Identities
$\tan(\theta + \pi n) = \tan\theta$
$\cot(\theta + \pi n) = \cot\theta$

Periodic Identities
$\csc(\theta + 2\pi n) = \csc\theta$
$\sec(\theta + 2\pi n) = \sec\theta$

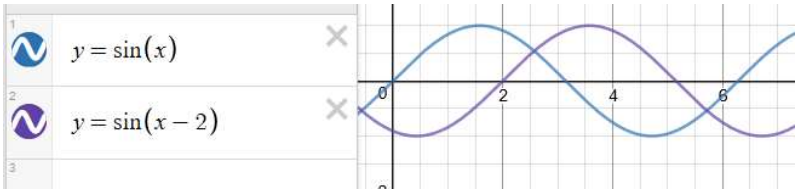
Horizontal & Vertical Shifts

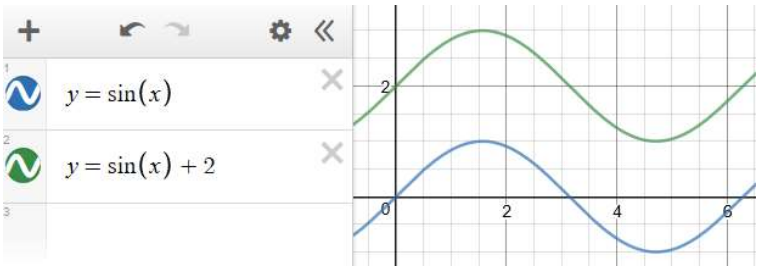
. . . You knew shifts were coming next . . .

As always, a horizontal shift is added or subtracted from the x in the middle of the function.

A vertical shift works the same as always, too. A vertical shift is added or subtracted at the end of the function.

Horizontal & Vertical Shifts





General Equation

$$y = a\, f(bx + c) + d$$

"f" is for the function:
sin, cos, tan, cot, sec, csc

a is the vertical stretch or compress factor
b is the horizontal stretch or compress factor
used to calculate the period
c is used to calculate the horizontal shift
d is the vertical shift

Phase Shift

This is the horizontal shift amount.
It is calculated from the general equation:
$y = a\, f(bx + c) + d$

Phase shift = -c/b

Because the horizontal shift is what is added to just 1x,
not to x times something other than one.

So we are dividing each term in the parentheses by b
so x has a coefficient of 1

Phase Shift

This is the horizontal shift amount.
It is calculated from the general equation:
y = a f(bx + c) + d

Phase shift = -c/b

For example, y = sin(2x+3)
Adjust this to sin(x + 3/2)
to see that the horizontal shift
is 3/2 to the left
OR just do –c/b to get a phase shift of -3/2.

Phase Shift & Vertical Shift
Stretch & Compress

y = a f(bx + c) + d

TRY: 3 sin (4x – 5π) + 6

Amplitude = ?
Period =
Frequency =
Phase Shift =
Vertical Shift =

Phase Shift & Vertical Shift
Stretch & Compress

$y = a\, f(bx + c) + d$

TRY: $3 \sin(4x - 5\pi) + 6$

Amplitude = $a = 3$
Period = ?
Frequency =
Phase Shift =
Vertical Shift =

Phase Shift & Vertical Shift
Stretch & Compress

$y = a\, f(bx + c) + d$

TRY: $3 \sin(4x - 5\pi) + 6$

Amplitude = $a = 3$
Period = standard period / $b = 2\pi/4 = \pi/2$
Frequency = ?
Phase Shift =
Vertical Shift =

Phase Shift & Vertical Shift
Stretch & Compress

$y = a\, f(bx + c) + d$

TRY: $3 \sin (4x - 5\pi) + 6$

Amplitude = 3
Period = $2\pi/4 = \pi/2$
Frequency = reciprocal of period = $2/\pi$
Phase Shift = ?
Vertical Shift =

Phase Shift & Vertical Shift
Stretch & Compress

$y = a\, f(bx + c) + d$

TRY: $3 \sin (4x - 5\pi) + 6$

Amplitude = 3
Period = $2\pi/4 = \pi/2$
Frequency = $2/\pi$
Phase Shift = $-c/b = -(-5\pi)/4 = +5\pi/4$ right
Vertical Shift = ?

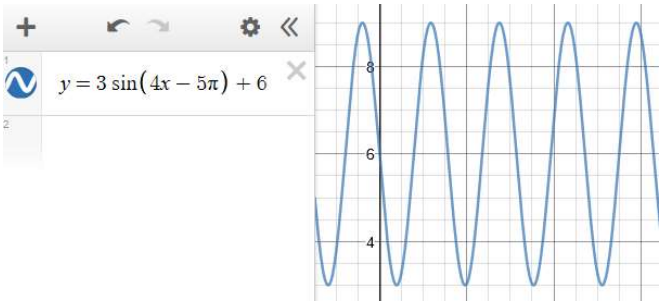
Phase Shift & Vertical Shift
Stretch & Compress
y = a f(bx + c) + d

TRY: 3 sin (4x – 5π) + 6

Amplitude = 3
Period = 2π/4 = π/2
Frequency = 2/π
Phase Shift = -(-5π)/4 = +5π/4 right
Vertical Shift = d = 6 up

Phase Shift & Vertical Shift
Stretch & Compress
y = a f(bx + c) + d

See: 3 sin (4x – 5π) + 6



Phase Shift & Vertical Shift
Stretch & Compress

$y = a\, f(bx + c) + d$

TRY: $-3 \csc(1/2x - \pi) + 1$

Amplitude = ?
Period =
Frequency =
Phase Shift =
Vertical Shift =

Phase Shift & Vertical Shift
Stretch & Compress

$y = a\, f(bx + c) + d$

TRY: $-3 \csc(1/2x - \pi) + 1$

Amplitude = none, it's csc … but it is vertically stretched + reflected
Period = ?
Frequency =
Phase Shift =
Vertical Shift =

Phase Shift & Vertical Shift
Stretch & Compress

$y = a\, f(bx + c) + d$

TRY: $-3 \csc(1/2x - \pi) + 1$

Amplitude = none, it's csc
Period = $2\pi/(1/2) = 4\pi$
Frequency = ?
Phase Shift =
Vertical Shift =

Phase Shift & Vertical Shift
Stretch & Compress

$y = a\, f(bx + c) + d$

TRY: $-3 \csc(1/2x - \pi) + 1$

Amplitude = none, it's csc
Period = $2\pi/(1/2) = 4\pi$
Frequency = $1/\text{period} = 1/4\pi$
Phase Shift = ?
Vertical Shift =

Phase Shift & Vertical Shift
Stretch & Compress

$y = a\, f(bx + c) + d$

TRY: $-3 \csc(1/2x - \pi) + 1$

Amplitude = none, it's csc
Period = $2\pi/(1/2) = 4\pi$
Frequency = $1/\text{period} = 1/4\pi$
Phase Shift = $-c/b = -(-\pi)/(1/2) = +2\pi$ right
Vertical Shift = ?

Phase Shift & Vertical Shift
Stretch & Compress

$y = a\, f(bx + c) + d$

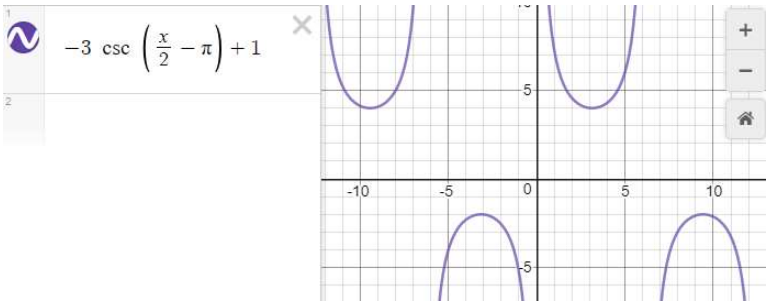
TRY: $-3 \csc(1/2x - \pi) + 1$

Amplitude = none, it's csc
Period = $2\pi/(1/2) = 4\pi$
Frequency = $1/\text{period} = 1/4\pi$
Phase Shift = $-c/b = -(-\pi)/(1/2) = +2\pi$
Vertical Shift = +1

Phase Shift & Vertical Shift
Stretch & Compress

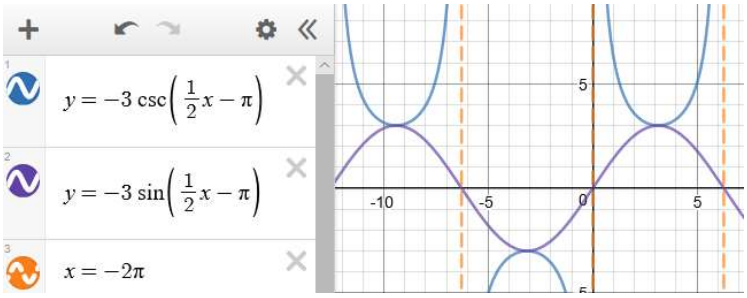
y = a f(bx + c) + d

Graph: -3 csc (1/2x – π) + 1



Asymptote Lines

Tip: For sec & csc, check the where the reciprocal crosses its mid-line . . .



Questions??

Review the Key Terms and Key Concepts documents for this unit.

Look up the topic at khanacademy.org and virtualnerd.com

Check our class website at nca-patterson.weebly.com

*Reserve a time for a call with me at jpattersonmath.youcanbook.me

We can use the LiveLesson whiteboard to go over problems together!