UNIT 2 LESSONS 6-7

PRECALCULUS B

LESSONS:

- Writing Equations of Trigonometric Functions
- Modeling with Trigonometric Functions

. . . . . We'll be starting with the graph or create the graph from the data points!

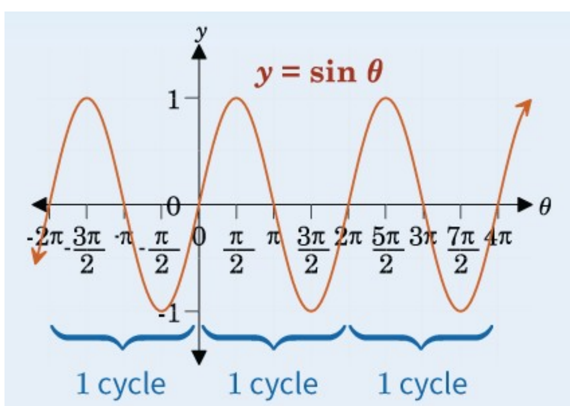
FIRST, REVIEW THE GRAPHS OF THE PARENT FUNCTIONS

*Take note of the basic shape, the intercepts, and the periods for each.

*We need to know the starting points before we look for any stretch or shift.

Sine	Cosecant
Cosine	Secant
Tangent	Cotangent

Sine



Sine

At the y-axis, this starts at the mid-line.

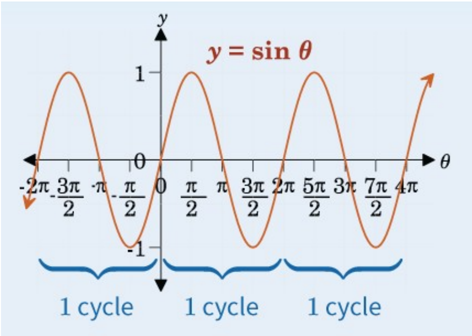
It goes up first.

The period is 2π.

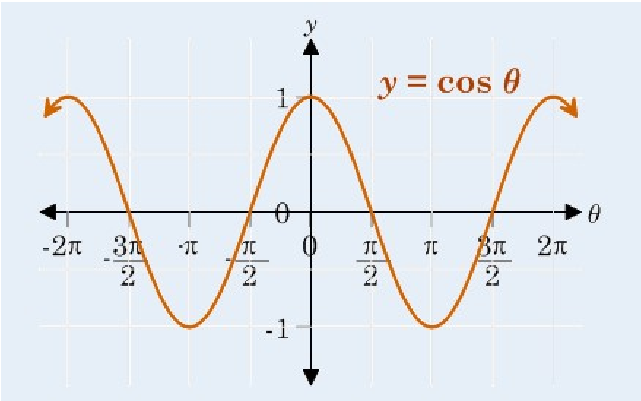
The middle of the cycle is at 1π.

The max & min are at π/2 & 3π/2.

The amplitude is 1.



Cosine



Cosine

At the y-axis, this starts at the maximum.

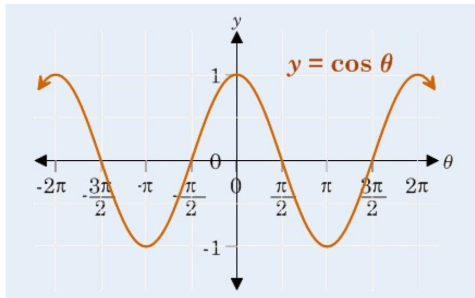
It goes down first.

The period is 2π.

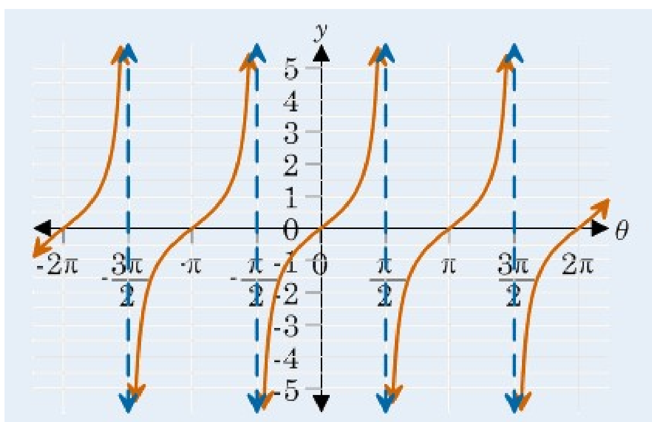
The middle of the cycle is at 1π.

The mid-line points are at π/2 & 3π/2.

The amplitude is 1.



Tangent



Tangent

At the y-axis, this starts at the turning point.

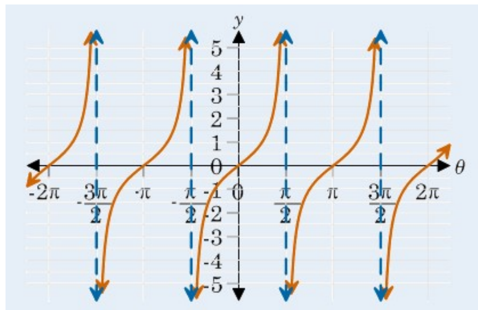
It goes down upward from left to right

The period is 1π.

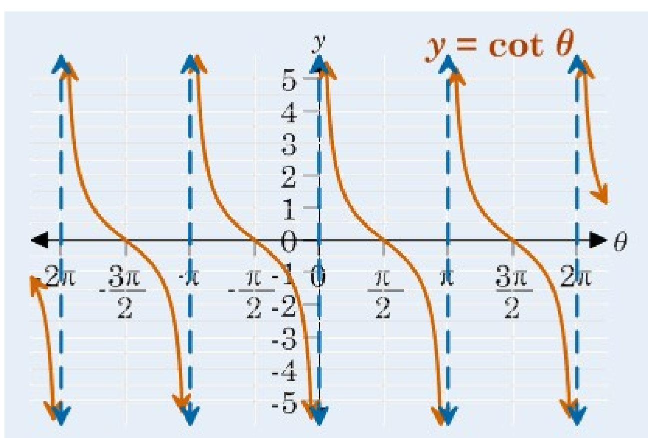
The middle of the cycle is at 0.

The asymptotes are at multiples of π/2 & -π/2.

There is no amplitude, but it can be stretched.



Cotangent



Cotangent

At the y-axis, there is an asymptote.

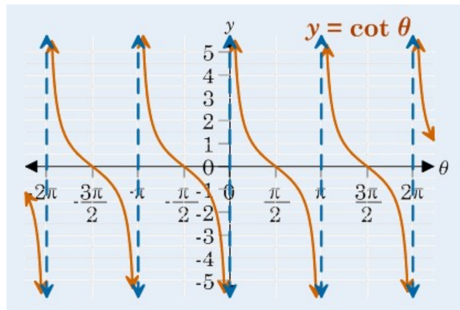
It goes down downward from left to right

The period is 1π.

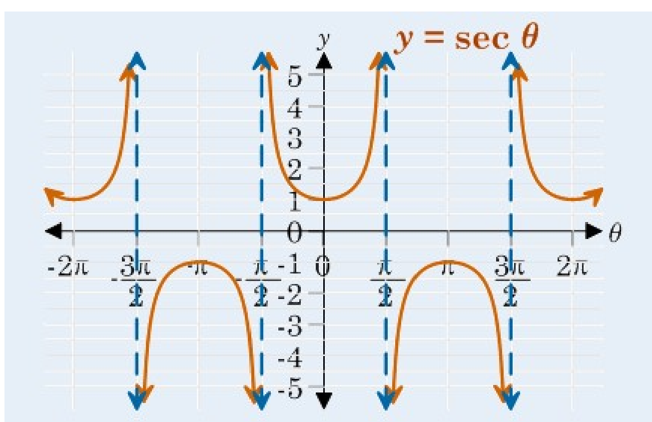
The middle of the cycle is at π/2.

The asymptotes are at 0 and multiples of π.

There is no amplitude, but it can be stretched.



Secant



Secant

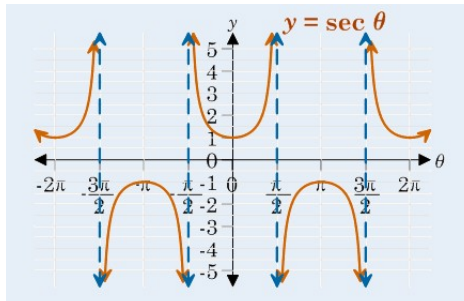
The turning point of the upper section is at 1 on the y-axis.

The turning point of the next lower section is at -1 centered below π.

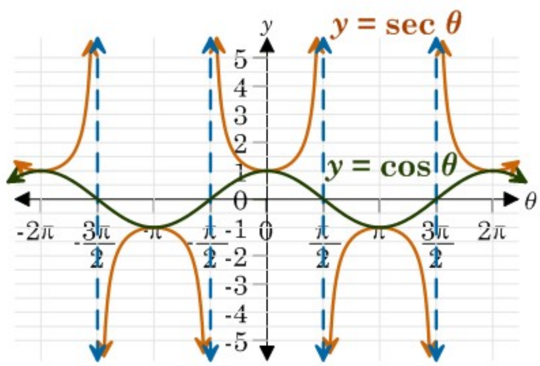
The period is 2π, like its reciprocal cosine.

The asymptotes are at multiples of π/2.

There is no amplitude, but it can be stretched.

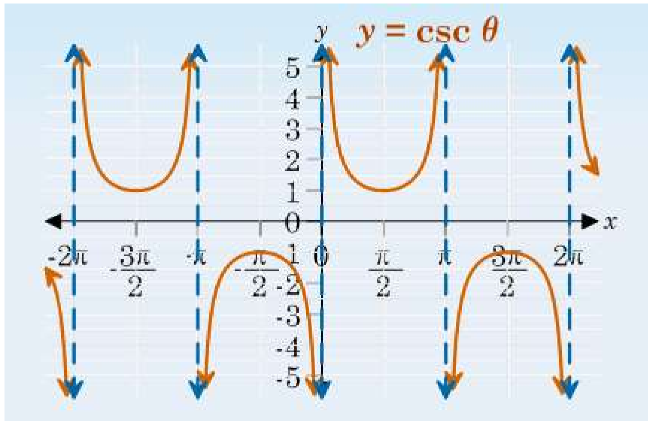


Secant



The turning points of secant are the same as the max/min points of cosine, its reciprocal.

Cosecant



Cosecant

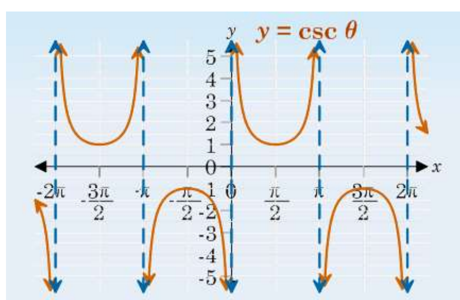
The turning point of the first upper section is at 1 and centered over π/2.

The turning point of the next lower section is at -1 centered below 3π/2.

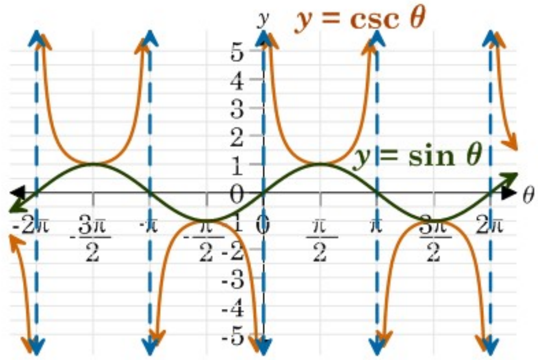
The period is 2π, like its reciprocal sine.

The asymptotes are at 0 and multiples of π.

There is no amplitude, but it can be stretched.



Cosecant



The turning points of cosecant are the same as the max/min points of sine, its reciprocal.

Last time we started with an equation with transformations, then identified the graph.

Now we are going to start with the graph, and identify the equation!

Steps for Writing an Equation of a Trigonometric Function

1. Identify the parent function and standard form of the transformed graph.
2. Locate the midline and identify the vertical shift.
3. Identify the amplitude.
4. Identify the period and use the formula for period to calculate the value of b.
5. Identify the phase shift and use the formula for phase shift with the value of b to find the value of c.
6. Substitute the values into the standard form of the equation.

Basic Equation Format

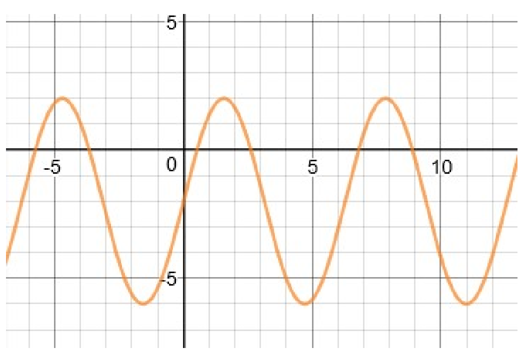
$y = a\ f(bx + c) + d$

"f" is for the function:
sin, cos, tan, cot, sec, csc

a is the vertical stretch or compress factor
b is the horizontal stretch or compress factor
c is the horizontal shift
d is the vertical shift

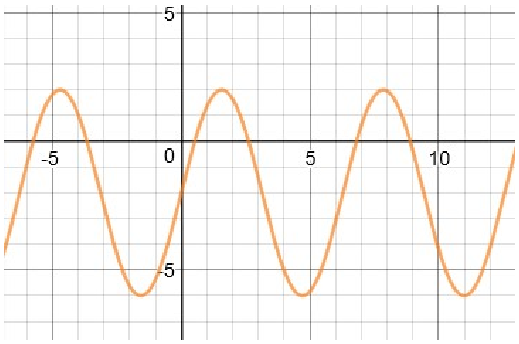
The Phase shift = -c/b

TRY:



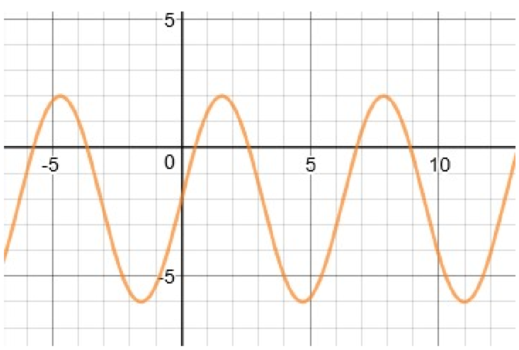
1st: Identify which parent function this looks like.

TRY:



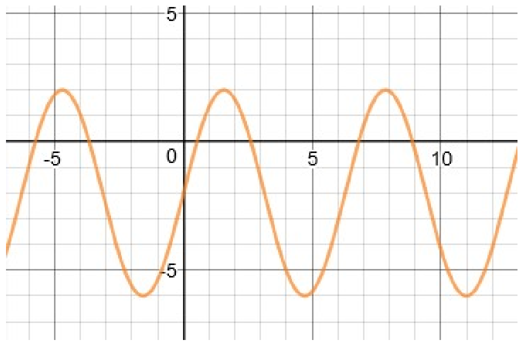
This could be sine shifted down, or, cosine shifted right and down. I will go with sine because one shift is simpler.

TRY:



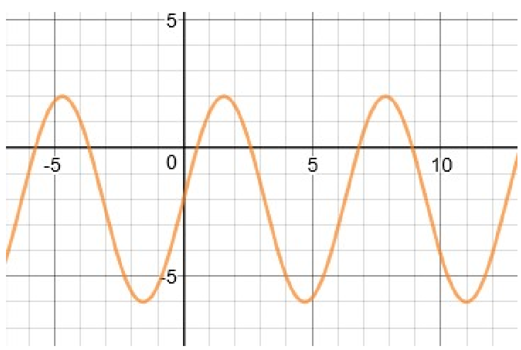
2nd: Locate the midline and the vertical shift.

TRY:



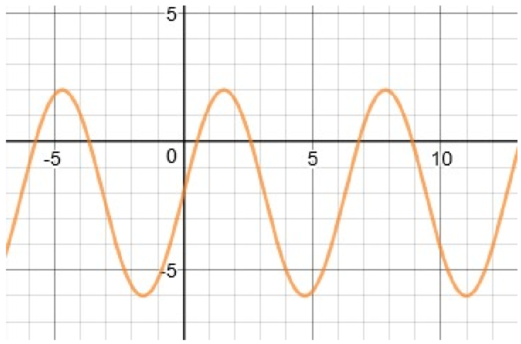
The midline (halfway between the max and min) is at -2.
The vertical shift is down 2, which goes in the spot for "d".

TRY:



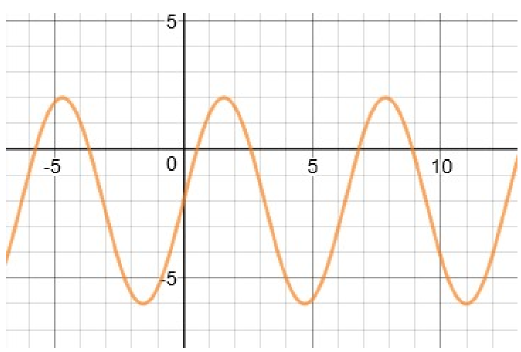
3rd: Identify the amplitude.

TRY:



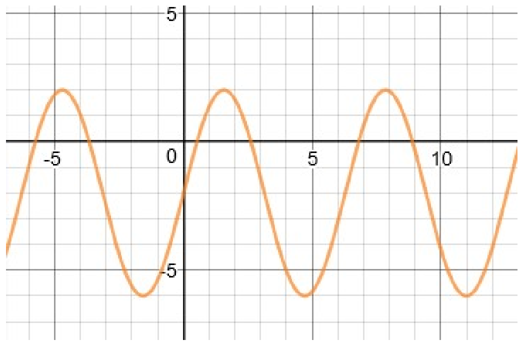
The amplitude is half of the distance between the max and min values. For this, the max is at 2 and the min is at -6, the difference is 8, so the amplitude is half of that, or 4. This will be "a".

TRY:



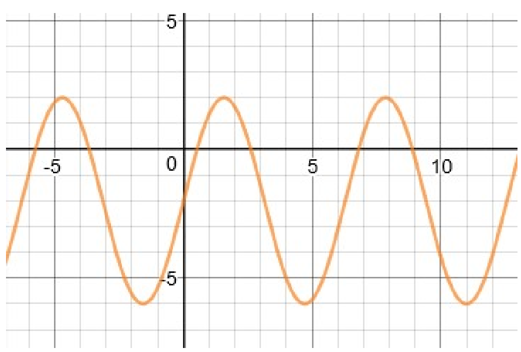
4th: Identify the period. The value for "b" is the parent function period divided by the period of the graph.

TRY:



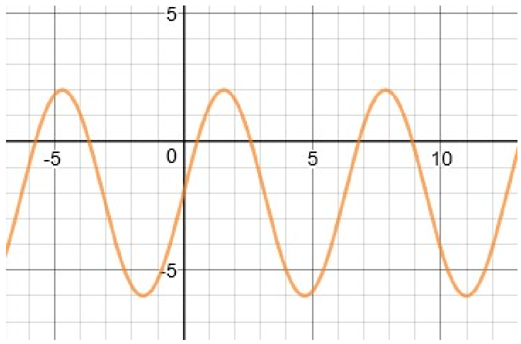
One cycle (look along the midline) goes from x=0 to x=2π (about 6.28). This is the same as the parent function. So, "b" is 2π/2π=1.

TRY:



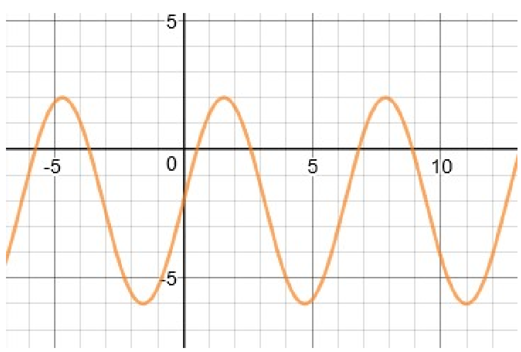
5th: Identify the phase shift. Look at any shift in the max point. Use this shift with the value you just got for b to find c.

TRY:



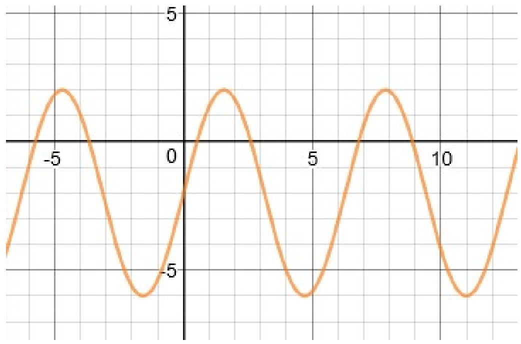
For basing this on sine, there is no horizontal phase shift. So, use 0 for the phase shift equation and get 0 = -c/b, or 0 = -c/1. So, c = 0

TRY:



6th: Fill in the equation!

TRY:

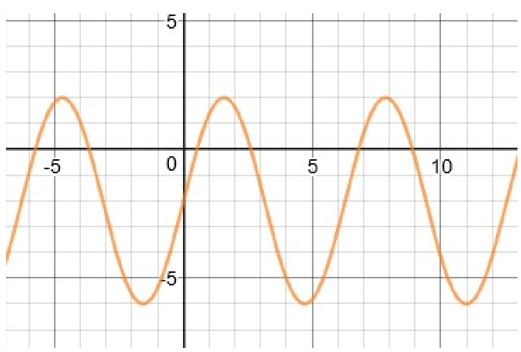


a=4, b=1, c=0, d= -2

y = 4 sin(1x+0) -2 … or …

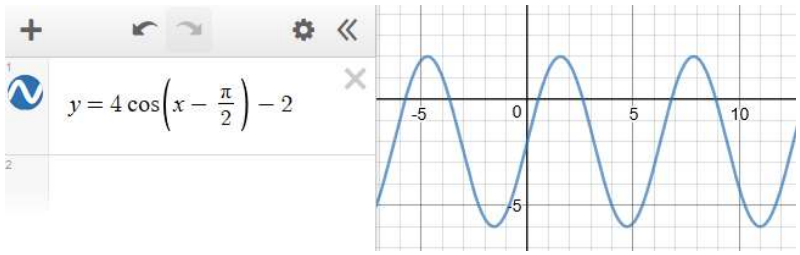
y = 4 sin(x) -2

TRY:



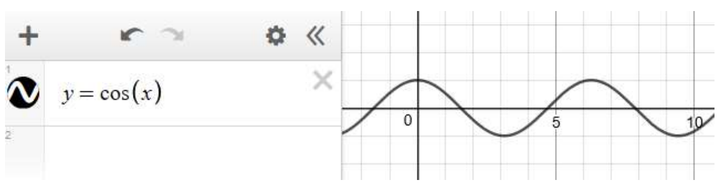
If we had started with cosine, instead, the only change would be a phase shift of π/2 (about 1.57).

TRY:

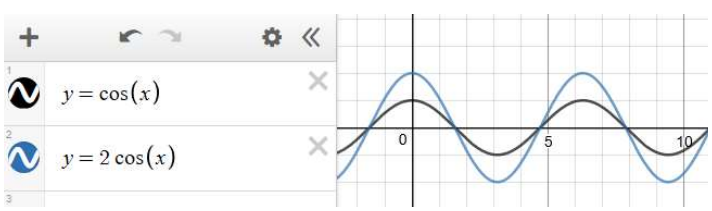


It's the same graph!

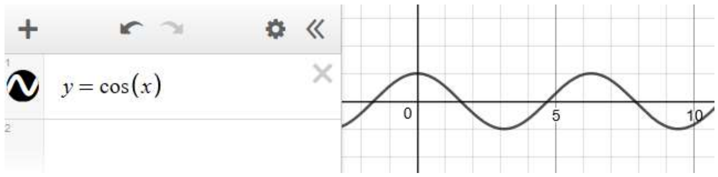
Watch another transformation:



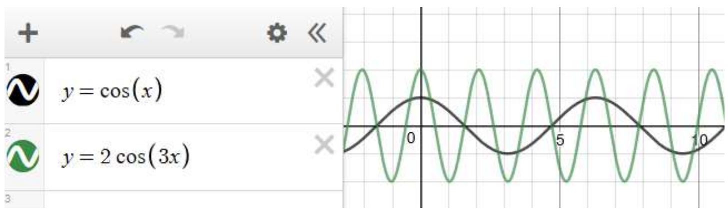
Change the amplitude . . . a=2



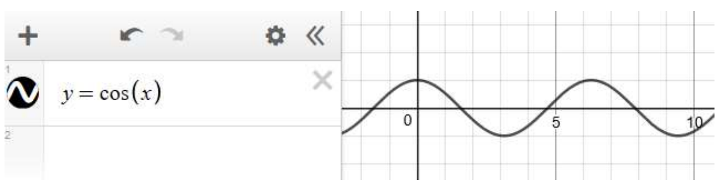
WATCH:



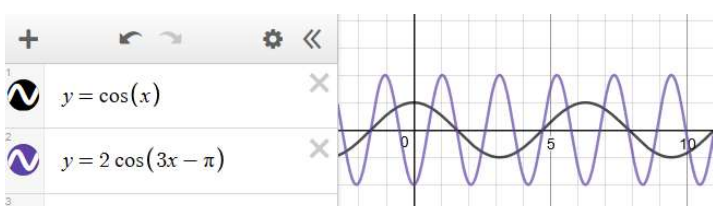
Change the period . . . b=3, so period=2π/3 (~2.1)



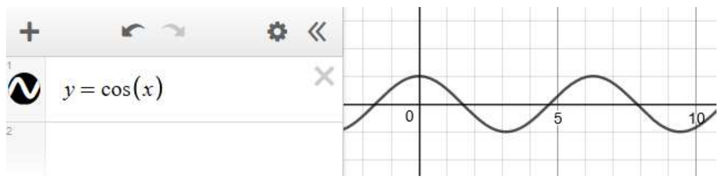
WATCH:



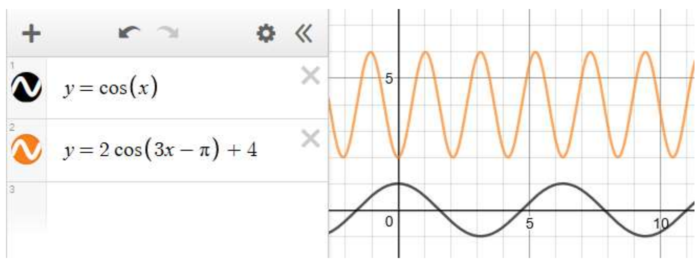
Add a phase shift . . . c= -π, phase shift = -c/b = π/3



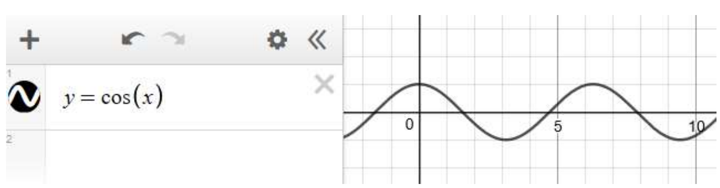
WATCH:



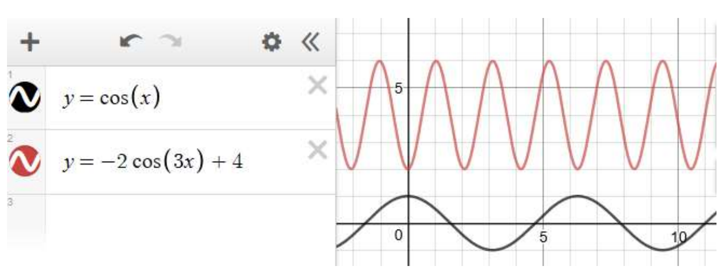
Then a vertical shift . . . d= 4



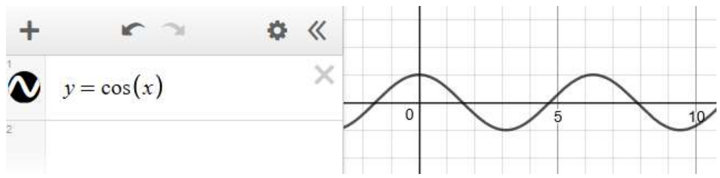
WATCH:



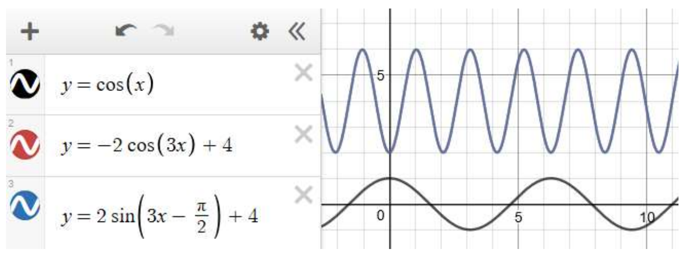
Or, same result with a reflection, instead of a phase shift!



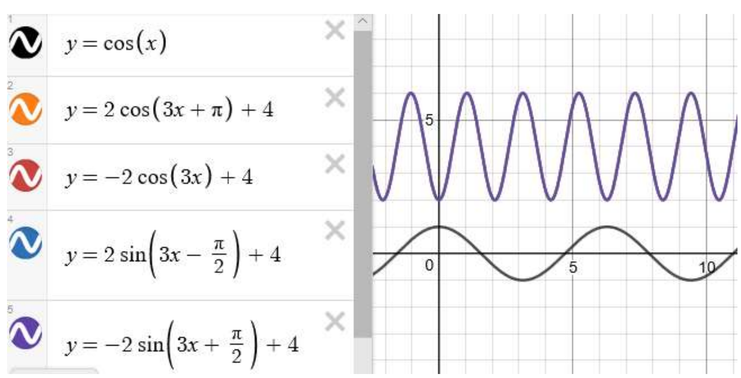
WATCH:



Or, the same result with sine and a shift!

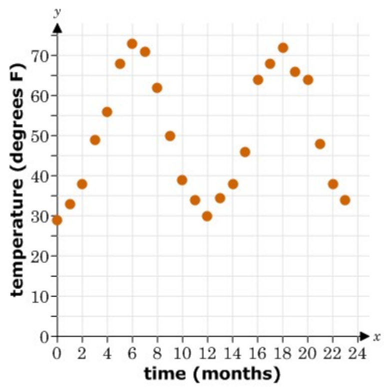


All correct!! All describe the same graph!!



So, as you work with the clues, check your equation with Desmos!!

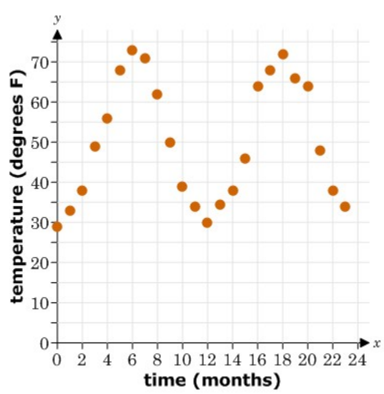
But real life data isn't a perfectly smooth curve . . .



This looks like a reflected cosine shifted up, but often in real life sine is preferred for the equation.

So, this could also be sine shifted right and up.

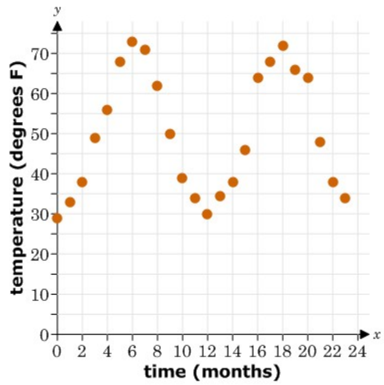
But real life data isn't a perfectly smooth curve . . .



We need values for a, b, c, & d, to fill in to:

$y = a \sin(bx + c) + d$

But real life data isn't a perfectly smooth curve . . .

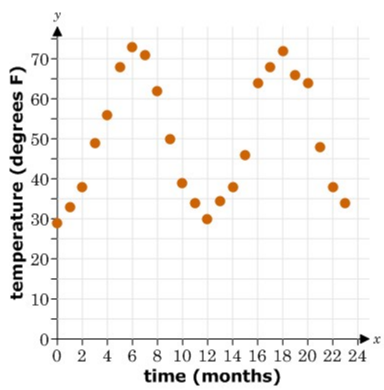


1st find a . . .

Amplitude is half the distance from max to min . . .

$$a = \frac{\text{maximum} - \text{minimum}}{2} = \frac{73-29}{2} = 22$$

But real life data isn't a perfectly smooth curve . . .

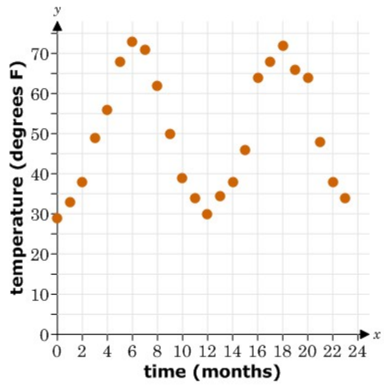


2nd find b . . .

The length of one cycle is 12.
b = parent period/graph period

$$b = \frac{2\pi}{12} = \frac{\pi}{6}$$

But real life data isn't a perfectly smooth curve . . .



3rd find c . . .

The phase shift = -c/b,
so c = -phase shift times b.

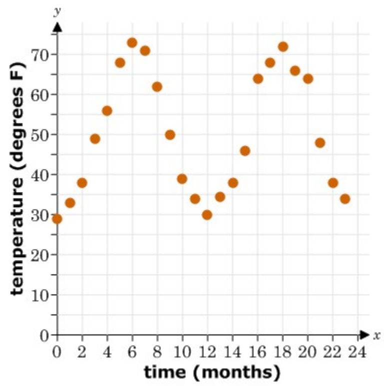
Look at the point on the midline, it shifted right from the y-axis by 3.

$$-\frac{c}{b}=3$$

$$-\frac{c}{\left(\frac{\pi}{6}\right)}=3$$

$$c=-\frac{\pi}{2}$$

But real life data isn't a perfectly smooth curve . . .



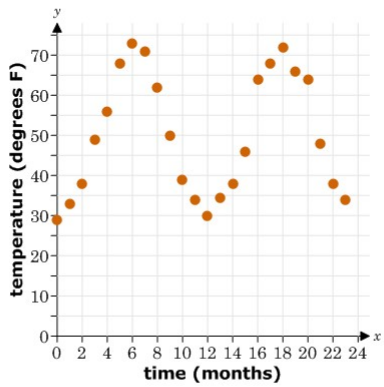
4th find d . . .

Look at the midline for the vertical shift from the x-axis.

Find the midpoint between the max and the min values.

$$d=\frac{\text{maximum}+\text{minimum}}{2}=\frac{102}{2}=51$$

But real life data isn't a perfectly smooth curve . . .

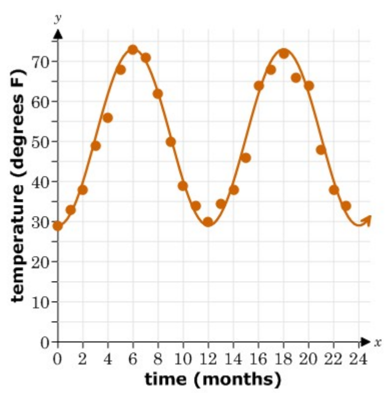


Finally, plug them all in . . .

$$f(x) = a\sin(bx + c) + d$$

$$f(x) = 22\sin\left(\frac{\pi}{6}x - \frac{\pi}{2}\right) + 51$$

But real life data isn't a perfectly smooth curve . . .



Finally, plug them all in . . .
And check with Desmos!

$$f(x) = a\sin(bx + c) + d$$

$$f(x) = 22\sin\left(\frac{\pi}{6}x - \frac{\pi}{2}\right) + 51$$

Questions??

Review the Key Terms and Key Concepts documents for this unit.

Look up the topic at khanacademy.org and virtualnerd.com

Check our class website at nca-patterson.weebly.com

*Reserve a time for a call with me at jpattersonmath.youcanbook.me

We can use the LiveLesson whiteboard to go over problems together!