UNIT 3 LESSON 1

PRECALCULUS B

LESSON 1:

- Trigonometric Identities
  - Reciprocal
  - Quotient
  - Pythagorean

What is an Identity in math?

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An identity is an equation that is always true, no matter what value you plug in!

For example:

a + 0 = a

(a)(1) = a

√a^2 = a

sin(a) = 1/csc(a)

RECIPROCAL IDENTITIES:

$$\sin\theta = \frac{1}{\csc\theta} \quad \csc\theta = \frac{1}{\sin\theta}$$

$$\cos\theta = \frac{1}{\sec\theta} \quad \sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{1}{\cot\theta} \quad \cot\theta = \frac{1}{\tan\theta}$$

. . . You already know these, and we've used them multiple times. ☺

QUOTIENT IDENTITIES:

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

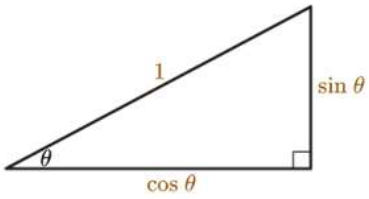
. . . These come from simplifying the basic ratios . . .

$$\frac{\sin\theta}{\cos\theta} = \frac{\left(\frac{\text{opposite}}{\text{hypotenuse}}\right)}{\left(\frac{\text{adjacent}}{\text{hypotenuse}}\right)} = \frac{\text{opposite}}{\text{adjacent}} = \tan\theta$$

PYTHAGOREAN IDENTITIES:

For right triangles in the Unit Circle,

- the hypotenuse is always 1,
- the side opposite is the sine of the central angle,
- the side adjacent is the cosine of the central angle.



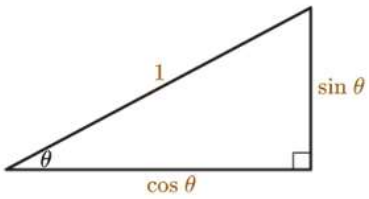
PYTHAGOREAN IDENTITIES:

So, by the Pythagorean Theorem:

$(\sin \theta)^2 + (\cos \theta)^2 = 1^2$

Or, more simply:

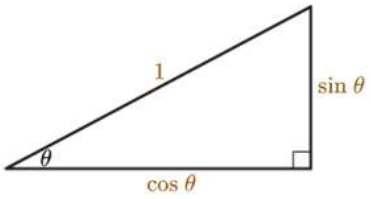
$\sin^2(\theta) + \cos^2(\theta) = 1$



PYTHAGOREAN IDENTITIES:

$\sin^2(\theta) + \cos^2(\theta) = 1$

We can get two other versions of this identity by dividing every term by either sine or cosine.



PYTHAGOREAN IDENTITIES:

sin^2(θ) + cos^2(θ) = 1

We can get two other versions of this identity by dividing every term by either sine or cosine.

SIMPLIFY THESE:

$$\frac{\sin^2\theta}{\sin^2\theta}+\frac{\cos^2\theta}{\sin^2\theta}=\frac{1}{\sin^2\theta}$$

$$\frac{\sin^2\theta}{\cos^2\theta}+\frac{\cos^2\theta}{\cos^2\theta}=\frac{1}{\cos^2\theta}$$

PYTHAGOREAN IDENTITIES:

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$$\frac{\sin^2\theta}{\cos^2\theta}+\frac{\cos^2\theta}{\cos^2\theta}=\frac{1}{\cos^2\theta}$$

$$\left(\frac{\sin\theta}{\sin\theta}\right)^2+\left(\frac{\cos\theta}{\sin\theta}\right)^2=\left(\frac{1}{\sin\theta}\right)^2$$

$$\left(\frac{\sin\theta}{\cos\theta}\right)^2+\left(\frac{\cos\theta}{\cos\theta}\right)^2=\left(\frac{1}{\cos\theta}\right)^2$$

$$1+\cot^2\theta=\csc^2\theta$$

$$\tan^2\theta+1=\sec^2\theta$$

PYTHAGOREAN IDENTITIES:

Pythagorean Identities
$\sin^2\theta+\cos^2\theta=1$
$1+\tan^2\theta=\sec^2\theta$
$1+\cot^2\theta=\csc^2\theta$

PYTHAGOREAN IDENTITIES:

Sometimes these are more useful when rearranged:

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$\cos^2\theta = 1 - \sin^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

$1 = \csc^2\theta - \cot^2\theta$

$\cot^2\theta = \csc^2\theta - 1$

$\tan^2\theta + 1 = \sec^2\theta$

$\tan^2\theta = \sec^2\theta - 1$

$1 = \sec^2\theta - \tan^2\theta$

The Fundamental Trig Identities:

Reciprocal Identities	Quotient Identities	Pythagorean Identities
$\sin\theta = \frac{1}{\csc\theta}$	$\tan\theta = \frac{\sin\theta}{\cos\theta}$	$\sin^2\theta + \cos^2\theta = 1$
$\csc\theta = \frac{1}{\sin\theta}$	$\cot\theta = \frac{\cos\theta}{\sin\theta}$	$1 + \tan^2\theta = \sec^2\theta$
$\cos\theta = \frac{1}{\sec\theta}$		$1 + \cot^2\theta = \csc^2\theta$
$\sec\theta = \frac{1}{\cos\theta}$		
$\tan\theta = \frac{1}{\cot\theta}$		
$\cot\theta = \frac{1}{\tan\theta}$		

We will use these to prove other identities . . .

How to Verify an Identity:

That is, show the substitutions and/or modifications that will convert the left side to match the right side!!

Verify that $\cos\theta/\sec\theta = \cos^2\theta$

$$\frac{\cos\theta}{\sec\theta} =$$

$$\cos\theta \cdot \frac{1}{\sec\theta} =$$

$$(\cos\theta)(\cos\theta) = \cos^2\theta$$

Verified!!

You Verify:

$\sin\theta \cot\theta \sec\theta = 1$

You Verify:

$\sin\theta \cot\theta \sec\theta = 1$

$\sin\theta \cot\theta \sec\theta =$

$\sin\theta \cdot \frac{\cos\theta}{\sin\theta} \cdot \frac{1}{\cos\theta} =$ substitute

You Verify:

$\sin\theta \cot\theta \sec\theta = 1$

$\sin\theta \cdot \cot\theta \cdot \sec\theta =$

$\sin\theta \cdot \frac{\cos\theta}{\sin\theta} \cdot \frac{1}{\cos\theta} =$ substitute

$\frac{\sin\theta \cdot \cos\theta \cdot 1}{1 \cdot \sin\theta \cdot \cos\theta} =$ multiply

$1/1 = 1$ cancel to reduce the fraction

Verified!!

You Verify:

$\cot^2 \theta - \csc^2 \theta = -1$

What identity has something that looks like this and uses these functions??

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What identity has something that looks like this and uses these functions??

Pythagorean Identity:

$1 + \cot^2 \theta = \csc^2 \theta$

We can substitute this for the $\csc^2 \theta$

You Verify:

$\cot^2 \theta - \csc^2 \theta = -1$

$\cot^2 \theta - \csc^2 \theta$

$= \cot^2 \theta - (1 + \cot^2 \theta)$

$= \cot^2 \theta - 1 - \cot^2 \theta$

$= (\cot^2 \theta - \cot^2 \theta) - 1$

$= 0 - 1$

$= -1$ **Verified!!**

Strategies:

1 – Express everything in terms of sine & cosine

2 – Substitute known formulas or identities . . . rearrange these as needed to match.

3 – Try factoring or distributing

4 – Use a common denominator to do any fraction addition or subtraction

5 – When a denominator has two terms, try multiplying top & bottom by the conjugate of the denominator . . . Remember, the conjugate is the same two terms, but reverse the sign in the middle

. . . See an example of each on page 5 of lesson 1

Strategies:

YES, you may need to try more than one strategy to finally get a way to make it work!!

It is like doing a puzzle or a maze . . .

So, be patient
and take your time!!

Working through the steps in the lesson examples will help train your brain to see possibilities for putting together these puzzles!

Questions??

Review the Key Terms and Key Concepts documents for this unit.

Look up the topic at khanacademy.org and virtualnerd.com

Come to Open Office time to ask me.
Check your Planner for the day & time.

Reserve a time for a call with me at jpattersonmath.youcanbook.me

We can use the LiveLesson whiteboard to go over problems together!