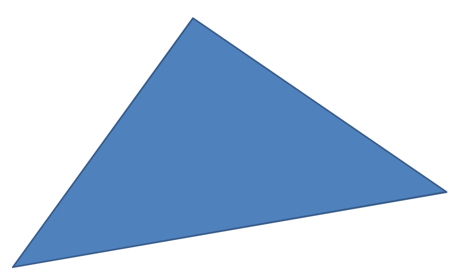
UNIT 3 LESSON 4
Law of Sines

PRECALCULUS B

The main idea in lessons 4 & 5 is Solving Triangles.

But what does it mean to "solve a triangle"???

To "solve a triangle" means to find the measurements of all three sides and all three angles!!!



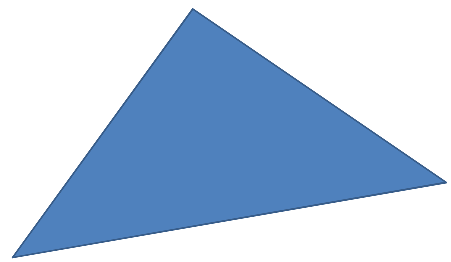
We can "solve a triangle" for all six amounts . . .

IF we are given at least three of them to start with

. . . Well, usually . . .

Starting possibilities are:

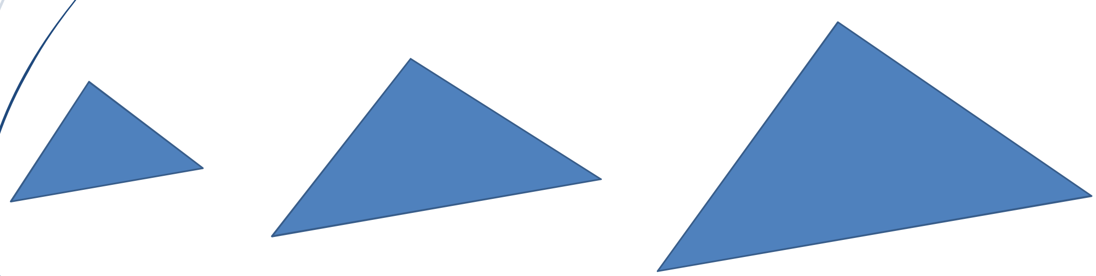
- 3 angles
- 2 angles & 1 side
- 1 angle & 2 sides
- 3 sides



AND, this is for ANY triangle,
not just right triangles!!

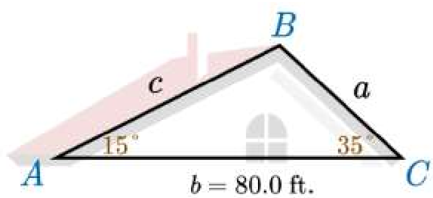
1st Case: Given 3 Angles – Can't be solved

If all angles are a fixed amount, but none of the sides are a fixed amount . . .
You can zoom in or out to change the length of the sides, but the size of the angles stay the same.



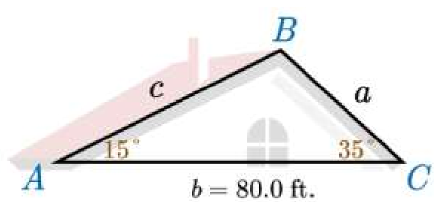
2nd Case: Given 2 Angles & 1 Side - ASA

ASA means two angles and the included side between them.
"Angle – Side – Angle"



NOTE: Angles are labeled with capital letters, sides with lower case letters.
And, the side opposite an angle will be the same letter.

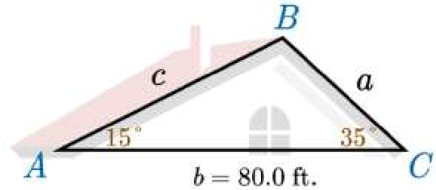
2nd Case: Given 2 Angles & 1 Side - ASA



We need to find the measurement of the third angle and the other two sides.

? NOTE: Any time you know two angles of a triangle you can quickly calculate the third angle . . . How??

2nd Case: Given 2 Angles & 1 Side - ASA

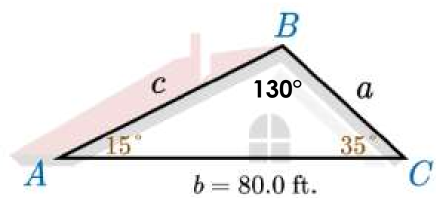


We need to find the measurement of
the third angle and the other two sides.

*The three angles of ANY triangle, always add up to 180°

angle B is $180-(15+35)=180-50=130°$

2nd Case: Given 2 Angles & 1 Side - ASA



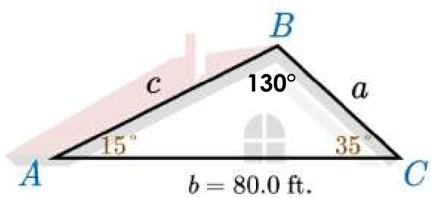
Now, to solve for the other two sides, set up
a proportion for each using the Law of Sines.

Law of Sines:

$$\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}.$$

NOTE: Using any two of the three ratios creates a proportion. And we know how to solve proportions!

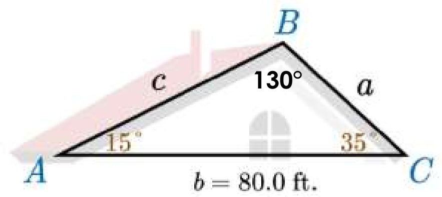
2nd Case: Given 2 Angles & 1 Side - ASA



Start with the ratio for the pair that you know the numbers for the angle and its opposite side.

$$\frac{\sin B}{b}$$

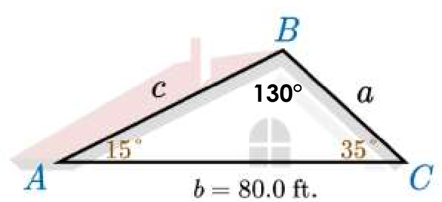
2nd Case: Given 2 Angles & 1 Side - ASA



Pick which side to solve for, then set up the proportion:

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

2nd Case: Given 2 Angles & 1 Side - ASA



Fill in the numbers you know . . . Then solve for the one that is left:

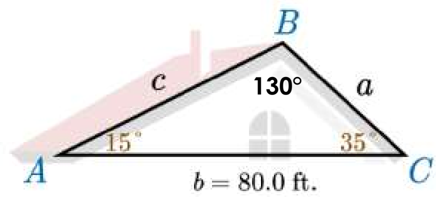
$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\frac{\sin 130°}{80} = \frac{\sin 15°}{a}$$

$$\frac{80 \sin 15°}{\sin 130°} = a$$

$$a \approx 27.0 \text{ ft.}$$

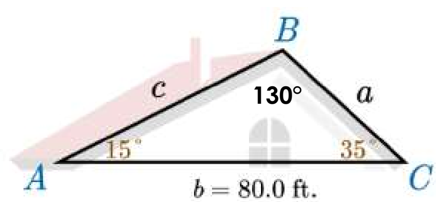
2nd Case: Given 2 Angles & 1 Side - ASA



TRY IT: Your turn to solve for the third side:

$$\frac{\sin B}{b} = \frac{\sin C}{c}$$

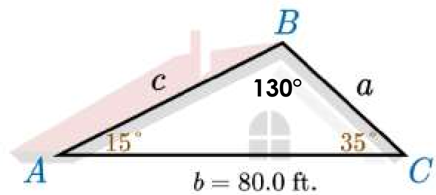
2nd Case: Given 2 Angles & 1 Side - ASA



TRY IT: Your turn to solve for the third side:

$$\frac{\sin B}{b} = \frac{\sin C}{c} \qquad \frac{\sin 130°}{80} = \frac{\sin 35°}{c}$$

2nd Case: Given 2 Angles & 1 Side - ASA



TRY IT: Your turn to solve for the third side:

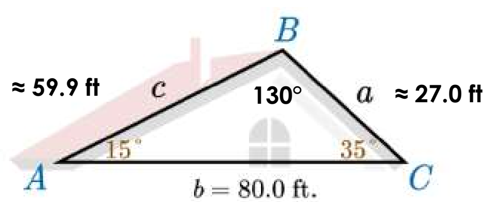
$$\frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin 130°}{80} = \frac{\sin 35°}{c}$$

$$\frac{80 \sin 35°}{\sin 130°} = c$$

$$c \approx 59.9 \text{ ft.}$$

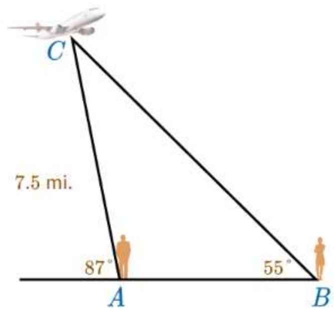
2nd Case: Given 2 Angles & 1 Side - ASA



The Triangle is Solved!

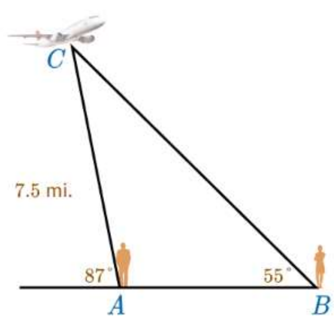
3rd Case: Given 2 Angles & 1 Side - AAS

AAS means two angles and the side not included between them.
"Angle – Angle – Side"



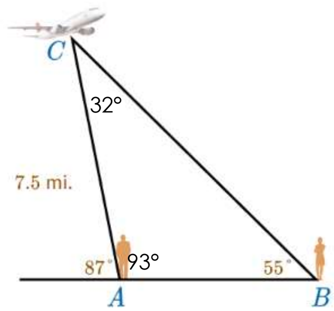
NOTE: In this example, the angle given on the left is NOT inside the triangle.
What do you do to get the measurement of the inside angle??

3rd Case: Given 2 Angles & 1 Side - AAS



The outside angle is supplementary to the inside angle,
so subtract from 180° to get the measure of the inside angle!!

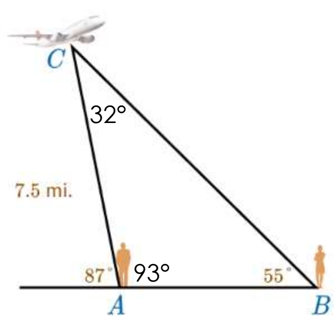
3rd Case: Given 2 Angles & 1 Side - AAS



This means the lower left inside angle is 180 – 87 = 93°
And now we can subtract the two inside angles from 180
to get the third inside angle: 180 – (93 + 55) = 32°

3rd Case: Given 2 Angles & 1 Side - AAS

Now that we have all three angles, back to the
Law of Sines to solve for the other two sides!

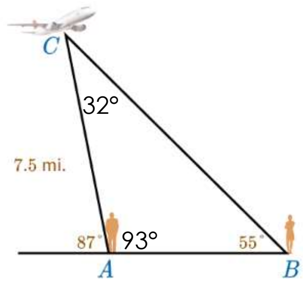


Start with the ratio of the pair you know . . .
In this case, we know the numbers for the pair of angle B and side b.

3rd Case: Given 2 Angles & 1 Side - AAS

TRY IT: Set up the two sets of proportions . . .

$$\frac{\sin B}{b} = \frac{\sin C}{c}$$



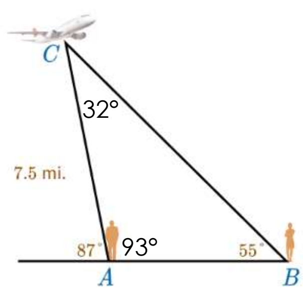
$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

3rd Case: Given 2 Angles & 1 Side - AAS

TRY IT: Set up the two sets of proportions . . .

$$\frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin 55°}{7.5} = \frac{\sin 32°}{c}$$



$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\frac{\sin 55°}{7.5} = \frac{\sin 93°}{a}$$

3rd Case: Given 2 Angles & 1 Side - AAS

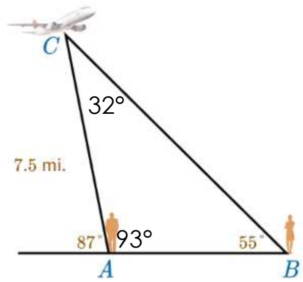
Now solve each proportion:

$$\frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin 55°}{7.5} = \frac{\sin 32°}{c}$$

$$\frac{7.5 \sin 32°}{\sin 55°} = c$$

$$c \approx 4.9 \text{ miles}$$



$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\frac{\sin 55°}{7.5} = \frac{\sin 93°}{a}$$

$$\frac{7.5 \sin 93°}{\sin 55°} = a$$

$$a \approx 9.1 \text{ miles}$$

The Triangle is Solved!

4th Case: Given 1 Angle & 2 Sides - SSA

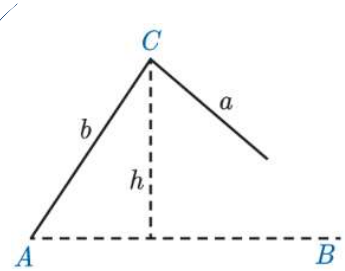
SSA means two sides and the not included angle.
"Side – Side – Angle"

**This is known as the Ambiguous Case!

Because the are 6 different ways this could turn out . . .

4^{th} Case: Given 1 Angle & 2 Sides - SSA

Given fixed lengths for sides a & b, and a fixed angle measurement for A . . .



If angle A is acute,
And side b is longer than side a,
So that angle C will be too high for side a to reach the bottom . . .

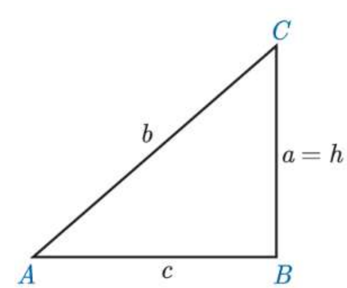
Then, NO triangle is possible to fit these numbers.

4^{th} Case: Given 1 Angle & 2 Sides - SSA

Given fixed lengths for sides a & b, and a fixed angle measurement for A . . .

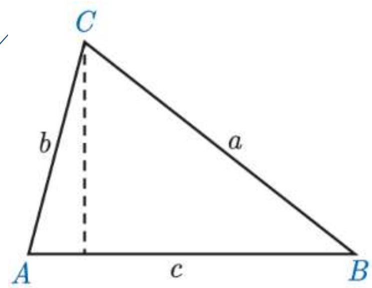
If angle A is acute,
And side b is longer than side a,
So that the height from angle C
Will be equal to side a . . .

Then, one triangle will fit these numbers . . .
And it will be a right triangle.



4th Case: Given 1 Angle & 2 Sides - SSA

Given fixed lengths for sides a & b, and a fixed angle measurement for A . . .



If angle A is acute,
And side a is longer than side b,
And longer than the height . . .

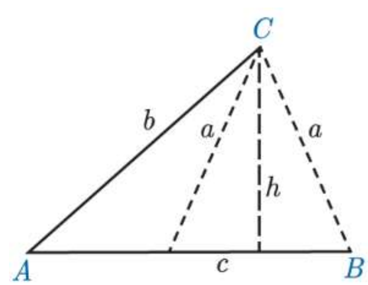
Then, one triangle will fit these numbers.

4th Case: Given 1 Angle & 2 Sides - SSA

Given fixed lengths for sides a & b, and a fixed angle measurement for A . . .

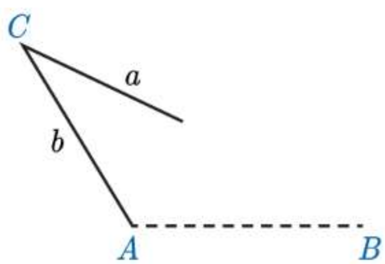
If angle A is acute,
And side a is shorter than side b,
But longer than the height . . .

Then, TWO triangles will fit these numbers.
Side a can land on either side of the height!



4th Case: Given 1 Angle & 2 Sides - SSA

Given fixed lengths for sides a & b, and a fixed angle measurement for A . . .



But if angle A is obtuse,
And side a is shorter than or equal to side b, . . .

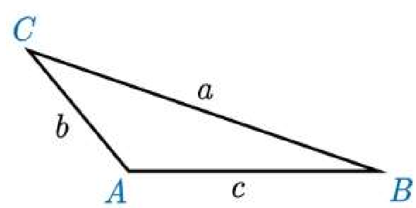
Then, NO triangle will fit these numbers.
Side a won't reach side c!

4th Case: Given 1 Angle & 2 Sides - SSA

Given fixed lengths for sides a & b, and a fixed angle measurement for A . . .

And if angle A is obtuse,
And side a is longer than side b . . .

Then, one triangle will fit these numbers.



4th Case: Given 1 Angle & 2 Sides - SSA

Given fixed lengths for sides a & b, and a fixed angle measurement for A . . .

When given SSA, always check how many, if any, solutions are possible!

If none, then there's no solving to do.

If two, then solve for both possibilities.

5th Case: Given 1 Angle & 2 Sides - SAS

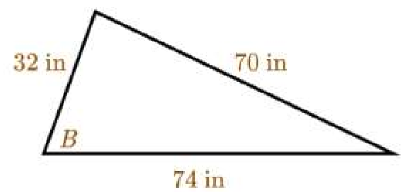
SAS means two sides and the included angle between them.
"Side – Angle – Side"

Notice that there is no ratio pair with both numbers given to start the Law of Sines.

So, we will need another new tool.
Law of Cosines … coming up in Lesson 5!

6th Case: Given 3 Sides - SSS

SSS means all three sides, but no angles.
"Side – Side – Side"



This is another case for the Law of Cosines!

Again, you don't have a known angle and opposite side pair to start the Law of Sines.

Questions??

Review the Key Terms and Key Concepts documents for this unit.

Look up the topic at khanacademy.org and virtualnerd.com

Come to Open Office time to ask me.
Check your Planner for the day and time.

*Reserve a time for a call with me at jpattersonmath.youcanbook.me

We can use the LiveLesson whiteboard to go over problems together!