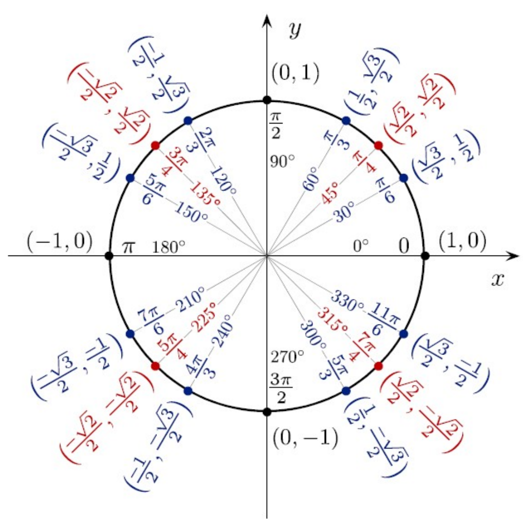
# UNIT 3 LESSON 6
# Trigonometric Equations

**PRECALCULUS B**

## We will need the Unit Circle:



## Solving Trig Equations . . .

For an equation of degree 1, with only one type of trig function, start like you would for solving any equation:

$$2\sin x - 1 = 0$$
$$2\sin x = 1$$
$$\sin x = \frac{1}{2}$$

Then use an inverse function for the final step:

$$\sin^{-1}(\sin x) = \sin^{-1}\left(\frac{1}{2}\right)$$
$$x = \sin^{-1}\left(\frac{1}{2}\right)$$
$$x = \frac{\pi}{6}$$

## Solving Trig Equations . . .

However . . .
Trig functions repeat, so list all possible angles!!

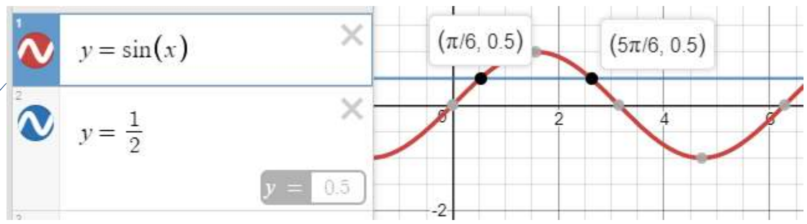
Check the unit circle, sin(x) = ½ at π/6 and at 5π/6!

And then you can go around the circle and repeat these every 2π!

$$x = \frac{\pi}{6} \pm 2n\pi \text{ and } x = \frac{5\pi}{6} \pm 2n\pi$$

## Solving Trig Equations . . .

You can also see this on Desmos!




For sin(x) = ½, Enter y = sin(x) & y = ½
The intersection shows the solutions at π/6 and at 5π/6!

## Solving Trig Equations . . .

Restrict the domain if you don't want all intersection points.

$2\sin x - 1 = 0$ over the interval $0 \le x \le 2\pi$

$$x = \frac{\pi}{6} \text{ and } x = \frac{5\pi}{6}$$

Instead of this solution if you don't restrict the domain:

$$x = \frac{\pi}{6} \pm 2n\pi \text{ and } x = \frac{5\pi}{6} \pm 2n\pi$$

## Solving Trig Equations . . .

For equations of degree 2 or
of mixed trig function types,
there are three main strategies:

- Factoring
- Squaring
- Quadratic Formula

And, sometimes you will need to do substitutions using trig identities to set it up for one of these strategies to work.

Often, when it is a degree 2, you will use a version of the Pythagorean identity

$\sin^2(x) + \cos^2(x) = 1$

## Solving Trig Equations . . . Factoring

Remember that sin(x) is a unit
you can factor like a plain x.

$$2\sin^2 x - \sqrt{3}\sin x = 0$$

$$\sin x(2\sin x - \sqrt{3}) = 0$$

Then use the zero product property:

$$\sin x = 0 \qquad\qquad 2\sin x - \sqrt{3} = 0$$

$$2\sin x = \sqrt{3}$$

$$\sin x = \frac{\sqrt{3}}{2}$$

## Solving Trig Equations . . . Factoring

Keep going and solve for x. For the exact angle, use the Unit Circle.

$$\sin x = 0 \qquad\qquad \sin x = \frac{\sqrt{3}}{2}$$

$$x = \sin^{-1} 0 \qquad\qquad x = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$x = 0 \qquad\qquad x = \frac{\pi}{3}$$

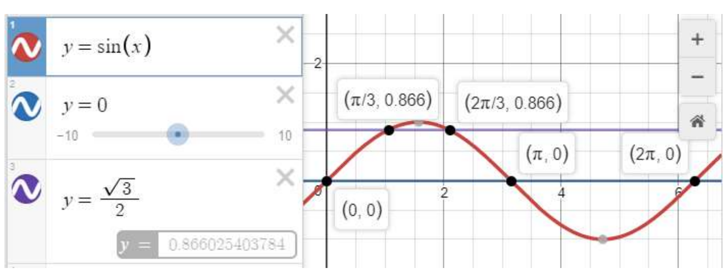
Check for other places in the domain interval where these are true,
but watch that you only use the quadrants that have the same sign for that function!

## Solving Trig Equations . . . Factoring

For the interval from 0 to 2π:

$$\sin x = 0 \qquad\qquad \sin x = \frac{\sqrt{3}}{2}$$

$$x = 0,\ x = \frac{\pi}{3},\ x = \pi,\ x = \frac{2\pi}{3},\ \text{and } x = 2\pi.$$



## Solving Trig Equations . . . Squaring

When you don't have a common factor, but you do have co-functions, try squaring both sides.

$$\sqrt{3}\sin x = \cos x$$

$$\left(\sqrt{3}\sin x\right)^2 = \left(\cos x\right)^2$$

$$3\sin^2 x = \cos^2 x$$

This gets you set up to substitute a Pythagorean identity like: $\cos^2 x = 1 - \sin^2 x$

$$3\sin^2 x = 1 - \sin^2 x$$

$$4\sin^2 x = 1$$

$$\sin^2 x = \frac{1}{4}$$

Now you only have one kind of trig function!
Next, solve for x . . .

## Solving Trig Equations . . . Squaring

$$\sin^2 x = \frac{1}{4}$$

$$\sin x = \pm\sqrt{\frac{1}{4}}$$

$$\sin x = \pm\frac{1}{2}$$

$$\sin^{-1}\left(\sin x\right) = \sin^{-1}\left(\pm\frac{1}{2}\right)$$

$$x = \pm\frac{\pi}{6}$$

Remember, a square root technically has 2 solutions!

$$x = \frac{\pi}{6},\ x = \frac{5\pi}{6},\ x = \frac{7\pi}{6},\ \text{and}\ x = \frac{11\pi}{6}.$$

Check for any other angles
where sin x = ½
in the interval 0 to 2π,
positive or negative this time!

## Solving Trig Equations . . . Quadratic Formula or, Quadratic Factoring

$$\sin^2 x - \cos x = \cos^2 x$$

$$\left(1-\cos^2 x\right) - \cos x = \cos^2 x$$

$$1 - \cos x = 2\cos^2 x$$

Substitute a Pythagorean identity so all terms use the same trig function.

$$2\cos^2 x + \cos x - 1 = 0$$

You now have a quadratic equation!

$$(2\cos x - 1)(\cos x + 1) = 0$$

Remember, treat a cos(x) like a plain x when factoring!

$$2\cos x - 1 = 0 \qquad \cos x + 1 = 0$$

$$2\cos x = 1 \qquad \cos x = -1$$

$$\cos x = \frac{1}{2}$$

Next, solve for x over the interval from 0 to 2π.

## Solving Trig Equations . . . Quadratic Formula or, Quadratic Factoring

$$(2\cos x - 1)(\cos x + 1) = 0$$

$$2\cos x - 1 = 0 \qquad \cos x + 1 = 0$$

$$2\cos x = 1 \qquad \cos x = -1$$

$$\cos x = \frac{1}{2}$$

$$\cos^{-1}(\cos x) = \cos^{-1}\left(\frac{1}{2}\right) \qquad \cos^{-1}(\cos x) = \cos^{-1}(-1)$$

$$x = \cos^{-1}\left(\frac{1}{2}\right) \qquad x = \cos^{-1}(-1)$$

$$x = \frac{\pi}{3} \qquad x = \pi$$

Solve for x over the interval from 0 to 2π.

Watch the quadrant signs!

$$x = \frac{\pi}{3},\ x = \pi, \text{ and } x = \frac{5\pi}{3}.$$

## Solving Trig Equations . . . a final note

In addition to always checking for all solutions within the given domain interval,

And checking which quadrants match the sign of what the trig function is equal to,

ALSO check for extraneous solutions!!

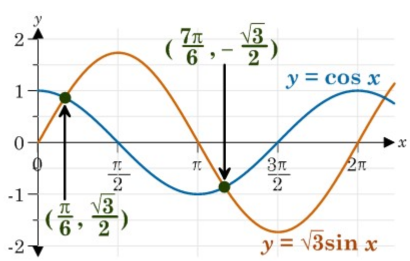
This often happens in the squaring method, or if you use the quadratic formula.

You can check by substitution, or by checking the intersection points of the graph!

## Solving Trig Equations . . . Extraneous Solutions

Example: $\sqrt{3}\sin x = \cos x$

$$x = \frac{\pi}{6},\ x = \frac{5\pi}{6},\ x = \frac{7\pi}{6},\ \text{and}\ x = \frac{11\pi}{6}.$$




But, only two of these are intersection points when it is graphed!
So the other two solutions are extraneous.

## Questions??

Review the Key Terms and Key Concepts documents for this unit.

Look up the topic at khanacademy.org and virtualnerd.com

Come to Open Office time to ask me.
Check your Planner for the day and time. 

*Reserve a time for a call with me at jpattersonmath.youcanbook.me

We can use the LiveLesson whiteboard to go over problems together!